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APPLICATION OF SEMIVARIOGRAMS AND KRIGING IN GEOTECHNICAL MODELLING

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Abstract. Semivariograms and kriging are very important tools for geotechnical modelling. The application of the recent results on these techniques, aided with computer software, yields new insights in different areas of science and engineering. In this paper, we use spherical semivariograms and ordinary kriging, for modeling of the specific geotechnical parameters in the coal deposit Brod-Gneotino. The obtained values for distribution of these parameters are quite convenient and effective for realistic slope analysis, exploration and long term planning of surface mines development. Although, in this paper we have case study, to proposed technique can be applied in general, whether it is an active open pit mine or design for a new one.

1. INTRODUCTION

The coal as an energetic mineral resource is from a fundamental importance in Macedonia, since it is the dominant source for electricity production. In Macedonia, coal deposits can be found in several so called sediment basins, characterized with tertiary geological age. Nowadays, only the deposits located in Pelagonical sediment basin are used for coal exploitation. More specifically, the coal is exploited at two surface mines in the Pelagonia basin: The so called open pits of “Podinska serija” in REK Bitola and the relatively recently activated open pit Brod-Gneotino.

Have in mind its importance, the coal deposit Brod-Gneotino has been subject of many geological, hydrogeological and geomechanical investigations and explorations, so there is a plenty data for detailed definition of the geology, hydrogeology and geomechanics of this deposit.

The application of the modern mathematical trends and computer software facilitates the process of interpretation of obtained data from the performed investigations, by creating of different types of geological and geotechnical datasets and models. Using the tools of spatial analysis, developed in the second half of the last century, aided by computer software, fast data processing can be performed. In comparison, the manual processing is quite complicated and time consuming.

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The computer software Leapfrog™ (more precisely its module Leapfrog Geo) is used for this study devoted on geotechnical modeling of the coal deposit Brod-Gneotino. With this geological modeling, 3D representation of the spatial structures of the deposit is performed. Likewise, with the geotechnical modeling, the measured data of geotechnical parameters is presented in 3D, and related to the defined geological structures.

3D modeling and spatial analysis of the collected data is being used in the mining for a quite long time. Before using modern computer into mathematical modeling, generating the 3D models, was made by using two dimensional specialized charts, cross-sections and diagrams. In the last three decades, the number of studies devoted on 3D modeling has rapidly increased, as a consequence of using specialized computer software. In this software, the modern 3D representations (models) are created with high resolution, and by using interpolating algorithms, [1].

The expansion and advancement of the 3D modeling in geology include integration of large amount of geological data, as well as additional available lithological, structural, geochemical, geophysical, geotechnical and other type of data. The constructed 3D representations can be used as interactive tools for exploring of mineral deposits, [6]. Three dimensional geological modeling (3DGM) is helpful for the geologists in quantitative analysis of three dimensional spatial structures, defining the spatial relations between geological objects. 3DGM technology gives us a technical support for drawing information and conclusions for the geology, 3D modeling and quantitative calculation of mineral resources in the deposits, [12].

Geotechnical modeling is fundamental in designing the mines with open pit or underground excavation. Fully defined and representative geotechnical model will provide information about the engineering and geological characteristics of the rock structure, defining its behavior in the domains of the mining. The model is composed of individual rock structures, showing similar geotechnical properties. The definition of this individual domains and its comprehensive approach is a keynote for the process of exploitation and related hazards, through facilitating the optimal solutions for the projects of the mines, [2], [8].

The development of the software improves the modeling, allowing the obtained models to be built in realistic 3D environment, by using implicit and semi implicit ways of modeling. The ways of modeling depend on the choice of data, application of certain trends and defining the geology, and additionally the obtained surfaces can be manually modified by the software's user. These options in the software are used for creating more complicated models, with renewing ability, by adding new data or new geological rules, and presenting more interpretations for large amount of data. The used mathematical models and software allow better understanding of the complexity of the deposits. We must be careful about the following things: the used data for modeling should be appropriate, representative and their quality should be assessed before we start with the mathematical modeling aided by computer software, [8].

The geotechnical model contributing to design of the mines should be based on comprehensive approach in geology, structures, alterations and erosions, and how they influence at engineering and geological characteristics of rock structures. The geotechnical engineers must have excellent understanding for the hazards and constraints of each separate model, as well as their influence of creating geotechnical domains, [8].

The mathematical modeling used here will be based on the theory from spatial analysis, or more precisely on semivariograms and ordinary kriging. Once a theoretical semivariogram is fixed, we are ready for spatial prediction. For spatial prediction, the geostatistics uses kriging. The term kriging is given in honor of the South African mining engineer, Daniel Gerhardus Krige. The question of expressing in a function the structure of spatial dependence or correlation, known in the geostatistics literature as structural analysis, is a key issue in the subsequent process of optimal prediction (kriging), as the success of the kriging methods is based on the functions yielding information about the spatial dependence detected. The functions referred to above are covariance functions (also called covariograms) and semivariograms, but they must meet a series of requisites, for example stationary and intrinsic hypothesis. As we only have the observed realization, in practice, the semivariograms derived from it may not satisfy such requisites. For this reason, one of the theoretical models (also called the valid models) that do comply must be fitted to it. Kriging aims to predict the value of a regularized function, $Z(\mathbf{s})$, at one or more non-observed points or blocks from a collection of data observed at n points (or blocks in the case of block prediction) of a domain D, and provides the best linear unbiased predictor (BLUP) of the regionalized variable under study at such non-observed points or blocks. Thus, the predictor support can be a point or a block, [9].

The paper is organized as follows. After the introduction, in Section 2, the theoretical foundations of this study is given, i.e. details on spherical semivariograms and ordinary kriging is presented. Section 3 is central part in this paper and is devoted on application of the methods of spatial analysis in geostatistical modeling of parameters important for slope stability and designing of surface mines. Conclusions of the paper are based on the previously obtained results of the case study Brod-Gneotino. The most important, the presented methods, approach and conclusions can be applied in general case of planning and designing of open pit and underground mining operations.

2. SEMIVARIOGRAMS AND KRIGING

In spatial analysis, the semivariogram is given by the following formula

$$\gamma(s_i - s_j) = \frac{1}{2} V(Z(s_i) - Z(s_j)),$$

for all $s_i, s_j \in D$, where D is continuous domain under study, and by $V(\cdot)$ denotes the variance of $\cdot$.

Under the second-order stationary hypothesis:

The random function $\{Z(s) : s \in D\}$ is said to be second-order stationary, if it has finite second-order moments and following hold:

a) The mathematical expectation exists and is constant, so it does not depend on the location s, $E(Z(s)) = \mu(s) = \mu$,

b) The covariance exists for every pair $Z(s)$ and $Z(s+h)$ depends only on the vector h joining the locations s and $s+h$, but not specifically on them, i.e. it holds

$$C(Z(s), Z(s+h)) = C(h), \text{ for all } s \in D \text{ and vectors } h;$$

and the intrinsic hypothesis (with no drift):

The random function $\{Z(s) : s \in D\}$ is said to be intrinsic if, for any given vector h of translation, the first-order increments $Z(s+h) - Z(s)$ are second-order stationary, i.e. $E(Z(s+h) - Z(s)) = \mu(s)$, where $\mu(s)$, the drift, is linear in h , and

$$C((Z(s+h) - Z(s)), (Z(s+h+h') - Z(s+h'))) = C(h, h'),$$

which is equivalent to

$$\frac{1}{2} V(Z(s+h) - Z(s)) = \gamma(h),$$

which is only a function of h,

it can be written as:

$$\gamma(h) = \frac{1}{2} V(Z(s+h) - Z(s)) = \frac{1}{2} E\left((Z(s+h) - Z(s))^2\right),$$

showing how the dissimilarity between $Z(s+h)$ and $Z(s)$ develops with distance h.

The semivariogram is the instrument used par excellence to describe the spatial dependence in the regionalized variable. The reason is that it covers a broader spectrum of regionalized variables than the covariance function, which is confined to second-order stationary random functions. This spectrum includes intrinsically stationary random functions, in which the covariance cannot be defined. In the second-order stationary framework, both the semivariogram and the covariogram are theoretically equivalent, [9]. Indeed,

$$C(h) = C(0) - \gamma(h).$$

In practice, the mean is unknown and it must be estimated from the data, which introduces a bias. The semivariogram of an intrinsically stationary random function depends on the vector h that connecting the locations (both on the distance between s and $s+h$, and also on the direction, but not on the locations themselves). Hence, in general terms, it is anisotropic. In the case when semivariogram depends only on distance, it is called isotropic.

The semivariogram is a non-decreasing monotone function, so that the variability of the first increments of the random function increases with distance. The semivariograms that correspond to second-order stationary random functions have a typical behavior at intermediate and large distances: They rise from the origin and increase monotonically with distance until approaching its limiting value, the a priori variance of the random function, $C(0)$, either exactly or asymptotically, [9].

The slope of the semivariogram indicates the change in the dissimilarity of the values of the regionalized variable with distance. The above-mentioned limiting value of the semivariogram is called the variance sill, or simply the sill (m), and the distance at which the sill is reached is termed the range, which defines the threshold of spatial dependence, i.e. the zone of influence of the random function. This means that, the range is the distance beyond which the values of the regionalized variable have no spatial dependence.

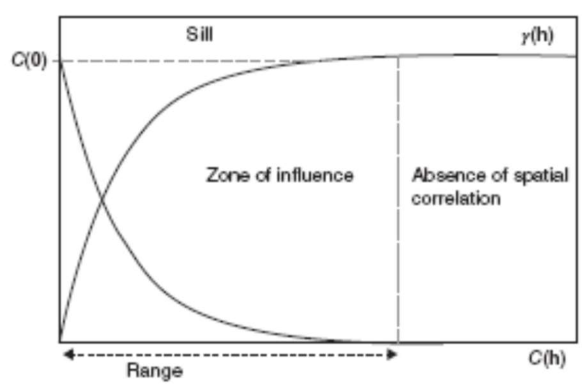

Figure 1: Example of Bounded semivariogram and its covariogram counterpart

Figure 1 illustrates that the larger the range, the larger is the zone of influence of the random function. In the case when the sill is reached asymptotically there is not a well-marked range, but a practical range (the distance at which the semivariogram takes the value $0,95m$). This practical range is closely related to the scale parameter, a, of the semivariogram (if the sill is reached exactly, i.e. in the case of a flat sill, a coincides with the range). Semivariograms that do not reach a sill are quite prevalent, and in particular this fact can occur when dealing with non-stationary random functions, in example when we have existence of a drift, intrinsically stationary random functions, or even second-order stationary random functions, in the case when the range exceeds the largest distance for which the semivariogram can be estimated.

Here we consider semivariogram that is associated with the second-order stationary hypothesis. Thus, it has a covariogram counterpart. This type of semivariograms also received the name of transition models because the distance at which the sill is reached represents the transition from the state of existence of spatial correlation to the state of absence of such spatial correlation, [9].

2.1. The spherical model

This model is valid on $\mathbb{R}^1$, $\mathbb{R}^2$ and $\mathbb{R}^3$. It is defined as

$$\gamma(|h|)=\begin{cases} m\left(1,5\dfrac{|h|}{a}-0,5\left(\dfrac{|h|}{a}\right)^3\right), & |h|\le a \\ m, & |h|>a \end{cases},$$

where $m=C(0)$ is the value of the semivariogram when it reaches the sill, and a is the range. The spherical semivariogram have a linear behavior near the origin, which indicates continuity, but a certain degree of irregularity in the random function. Also, it can be easily checked,

$$\frac{\partial\gamma(|h|)}{\partial h}=m\left(\frac{1,5}{a}-\frac{|h|^2}{a^3}\right),$$

hence the slope of the semivariogram at the origin is $1,5\dfrac{m}{a}$. The tangent at the origin intersects the sill at $|h|=\dfrac{2}{3}a$. Considering behavior at large distances, we have that it reaches the sill at $|h|=a$. This well-defined range, along with its simple polynomial expression and its validity on $\mathbb{R}^1$, $\mathbb{R}^2$ and $\mathbb{R}^3$, are some of the reasons for the wide use of the spherical semivariogram in practical applications. But the main reason is that an almost linear behavior up to a certain distance (the range) and then stabilization matches a large variety of observed regionalizations, [9].

Next, we will give the kriging equations, which give us a prediction of the value of the random function $Z(s)$ at a non-observed point (or block) as a linear combination of the values of the random functions at the sampled points (or blocks) or at a set of them that are close to the prediction point. Clearly, we will also give the expression of the prediction error variance, or simply the kriging variance, which indicates how accurate the kriging prediction is. These equations are obtained by imposing on the predictor the classical conditions of unbiasedness and minimum variance, i.e. by imposing on the prediction error zero expectation and minimum variance (that is, minimizing the mean squared prediction error). It is nevertheless necessary to make the following clarification: the minimization of the mean-squared prediction error stems from the assumption that the semivariogram is known. Usually, this is not the case in many practical situations, since the kriging prediction is based on the empirical semivariogram (to which a theoretical semivariogram is fitted to). Moreover, it is difficult to measure the consequences of not using the true semivariogram. More precisely, in the case of point observation support, the point kriging predictor $Z^*(s_0)$ at the non-observed point s_0 is given by

$$Z^*(s_0) = \sum_{i=1}^{n} \lambda_i Z(s_i),$$

where $Z(s_i)$ are the values observed at the n points in the neighborhood of s_0, the prediction point, and λ_i are the kriging weights obtained by imposing on the prediction error the classical conditions above referred. In many occasions we are interested in block prediction. That is, our aim is to predict the average value of the random function being studied in a block V, given by $Z_V(s)$, which is assigned to the point $s \in V$

$$Z_V(s) = \frac{1}{|V|} \int_V Z(s')\, ds',$$

where $|V|$ is the area or the volume of V, s' sweeps throughout V and s is a point in V to which the average value of the block is assigned.

In such cases, when the observations are based on points, the predictor of the average value of the random function in V is given by

$$Z^*(V) = \sum_{i=1}^{n} \lambda_i Z(s_i).$$

In the case of a block observation support, that is, the data available are the average values in blocks v_i, the block predictor in V is

$$Z^*_V = \sum_{i=1}^{n} \lambda_i Z_{v_i}(s_i),$$

where $Z_{v_i}(s_i)$ to be the average value in the block v_i, which is assigned to the point $s_i \in v_i$

$$Z_{v_i}(s_i) = \frac{1}{|v_i|} \int_{v_i} Z(s')\,ds',$$

as s' sweeps all over v_i, [9].

Clearly, the quality of kriging predictions is built on the size of the sample and the quality of the data, but it also confides in:

- the location of the realizations (if they are uniformly distributed in the domain under study, there will be better coverage and more information about what happens in that domain than if realizations are grouped);
- the distance between the points (or blocks) observed and the point or block to be predicted (more trust should be placed in nearby realizations than in distant realizations);
- the spatial continuity of the random function being studied (it is easier to predict the value of a regular random function at a point or over a block than of a random function that fluctuates markedly).

At the end of this section, let us make a remark that the main advantage of kriging over other spatial interpolation techniques (inverse distance method, splines, polynomial regression, among others) is that not only does it take into account geometric characteristics and the number and organization of locations, but also considers the structure of the spatial correlation that is deduced from the information available through semivariogram structures, hence yielding more reliable predictions. Therefore, the weights that kriging is using are not calculated on the basis of an arbitrary rule that can be used in some cases but not others, but rather on the behavior of the function that represents the structure of spatial correlation. In this sense, it is a more flexible method than those mentioned above, [9]. Additionally,

- Kriging makes it possible to measure how accurate are the predictions using the prediction error variance (the kriging variance) and can yield a map of the standard deviation of the prediction errors;
- kriging variance does not depend on the actual realization of the random function, which acts as a probability shelter, meaning we can calculate it before learning the values of the variables at those points, providing we know the structure of the spatial dependence of that random function. This is extremely useful when designing networks of optimum observations, that is, for selecting locations to be observed that provide the least kriging variance;

- kriging is an exact interpolator, which means that at the points that make up part of the sample, the kriging prediction corresponds with the value observed, the kriging variance therefore being zero. In short, the observation support can be points or blocks. In the first case the prediction support can be also points or blocks; in the second case, it can be only blocks, [9].

3. SEMIVARIOGRAMS AND KRIGING IN MODELING

In this section, we are going to use spherical semivariograms, in order to obtain realistic 3D representation of the certain geotechnical parameters on sand and coal in the coal deposit Brod-Gneotino. With these mathematical tools and the module Leapfrog Geo from the software Leapfrog^{TM}, the collected data [3]-[5] (see also [7], [10]-[11]), is analyzed and the following quasi-homogeneous domains in the Brod-Gneotino deposit are obtained: quasi-homogeneous domains of sand's volume weight, quasi-homogeneous domains of internal friction of sand determined by the direct shear, quasi-homogeneous domains of cohesion of sand determined by direct shear, quasi-homogeneous domains of the angle of internal friction of coal determined with triaxial tests and quasi-homogeneous domains of the angle of internal friction of coal determined with triaxial tests.

In the sequel, we are going to give detailed survey of the upper mentioned quasi-homogeneous domains.

3.1. Quasi-homogeneous domains of sand's volume weight

The data used for mathematical modeling of quasi-homogeneous domains belong to geological age of Quaternary, Pliocene and Miocene age, hence the created quasi-homogeneous domains will correspond to the part of geological model with Quaternary, Pliocene and Miocene age, without the part belonging on trepel and coal layers.

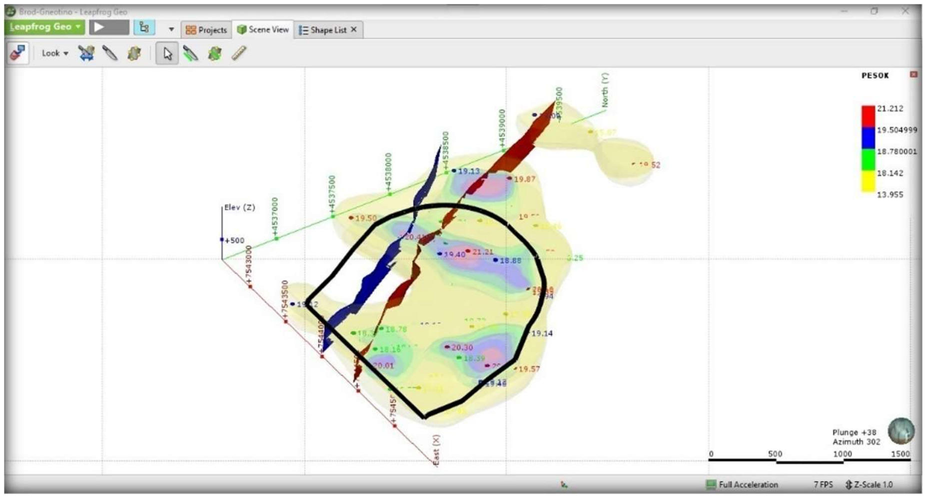

Figure 3: 3D representation of quasi-homogeneous domains of sand's volume weight

The obtained values for volume weight of the sand in the surface layer in Brod-Gneotino are divided on two parts:

- Gravelly sand (SP,SW) with **γ=20,10 [kN/m^3]**
- Silty sand (SFc, SFs)) with **γ=18,74 [kN/m^3].**

By the 3D representation of the quasi-homogeneous domains of the volume weight of the sand, we have the following:

- ✓ Data used for mathematical modeling of these domains are in the range from 13,95 to 21,21 [kN/m^3], with median **18,78 [kN/m^3]**,
- ✓ Domain with the highest values (19,50 – 21,21 [kN/m^3]) is located in the north part and in the part of east final slope of surface layer,
- ✓ The domains with lower values are more represented i.e. 13,95 – 18,14 [kN/m^3] and 18,14 – 18,78 [kN/m^3]),
- ✓ Since the obtained values belong in the intervals with lower values, we can consider the option, when analyzing the slope stability, to use lower values for volume weight of sand, compared to the obtained values.

3.2. Quasi-homogeneous domains of internal friction of sand determined by direct shear

Here, for mathematical modeling of quasi-homogeneous domains of the angle of internal friction of the sand, determined with direct shear, we use data belonging to geological layers with Pliocene and Miocene age, hence the created quasi-homogeneous domains will correspond to the part of geological model with Pliocene and Miocene age, without the part belonging on trepel and coal layers.

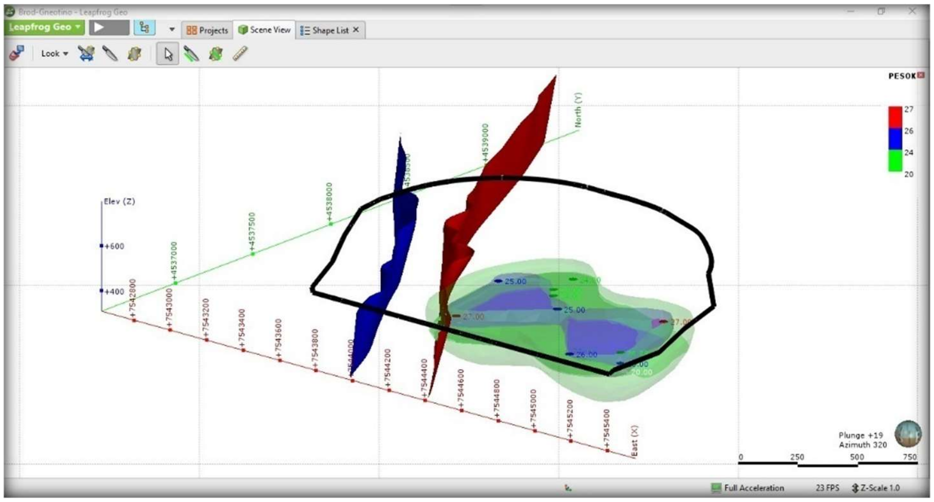

Figure 4: 3D representation of quasi-homogeneous domains of the angle of internal friction of the sand determined with direct shear

The obtained values for the angle of internal friction obtained with the direct shear of the sand in the surface layer in Brod-Gneotino are divided on two parts:

- Gravelly sand (SP,SW) with $\varphi = 28°$ and
- Silty sand (SFc, SFs)) with $\varphi = 17,65°$.

By the 3D representation of the quasi-homogeneous domains of the angle of internal friction obtained with direct shear of the sand, we have the following:

- ✓ Data used for mathematical modeling of these domains are in the range from $\varphi = 20°$ to $\varphi = 27°$, with median $\varphi = 24°$, located in the non-fault domain,
- ✓ Domain with the highest values ($26°$ - $27°$) is almost in punctate structure,
- ✓ The domains with lower values are more represented i.e. $20° - 24°$ and $24° - 26°$, and they are located in south-east part of the border of the surface mine,
- ✓ It can be noticed that the obtained values for this geotechnical parameter are located outside of the domains of separated quasi-homogeneous

domains. Since the domains with the values $20° - 24°$, the mentioned values can be used for analysis of slope stability of surface mine.

3.3. Quasi-homogeneous domains of cohesion of sand determined by direct shear

The data used for the mathematical modeling of quasi-homogeneous domains of the cohesion of sand determined by direct shear, belonging to geological layers with Pliocene and Miocene age, hence the created quasi-homogeneous domains will correspond to the part of geological model with Pliocene and Miocene age, without the part belonging on trepel and coal layers.

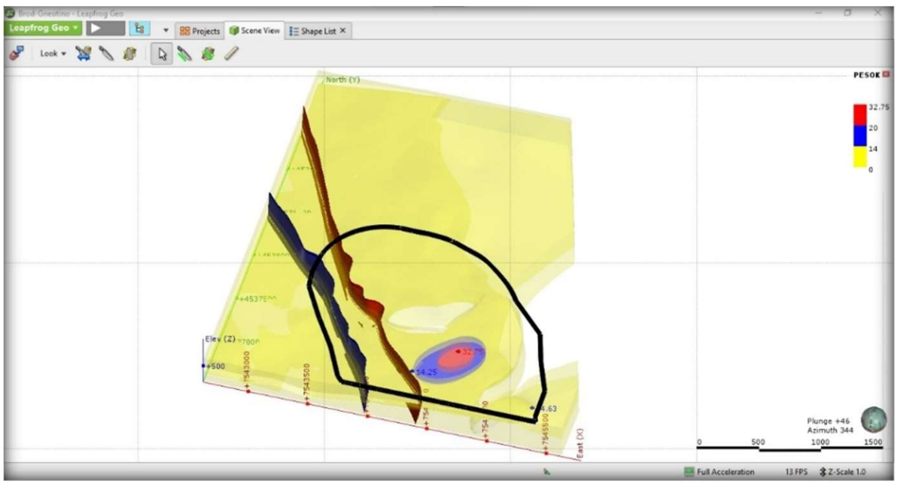

Figure 5: 3D representation of quasi-homogeneous domains of the cohesion of the sand determined with the direct shear

The obtained values for cohesion by direct shear of the sand in the surface mine in Brod-Gneotino are divided on two parts:

- Gravelly sand (SP,SW) with $c = 0,00[kN/m^3]$ and
- Silty sand (SFc, SFs)) with $c = 8,00[kN/m^3]$.

By the 3D representation of the quasi-homogeneous domains of the cohesion obtained with the direct shear of the sand, we derive the following conclusions:

- ✓ Data used for mathematical modeling of these domains are in the range from $c = 0,00[kN/m^3]$ to $c = 32,75[kN/m^3]$, with median $c = 0,00[kN/m^3]$, located in the non-fault domain,

- ✓ These quasi-homogeneous domains are formed by 13 numerical data, from whose only three of them are not zeros, giving the reason why modeling of the domains is not made like in the other geotechnical parts,
- ✓ The most representative domain with median value $c = 0,00[kN/m^3]$ is with correlation with the regular value of the cohesion of the sands,
- ✓ Non-zero values of the cohesion are $14,25$; $14,63$ and $32,75[kN/m^3]$. They are located in the medium part of the surface mine and they belong of the group of silty sands.
- ✓ The final slopes in the frame of the whole border of the surface mine are characterized with cohesion value $c = 0,00[kN/m^3]$.

3.4. Quasi-homogeneous domains of the angle of internal friction of coal determined with triaxial tests

The data used for modeling of the angle of the internal friction of the coal, determined by the triaxial test belong in the geological sections of the five coal layers with Miocene age, hence the created quasi-homogeneous domains will refer to these parts from the geological model.

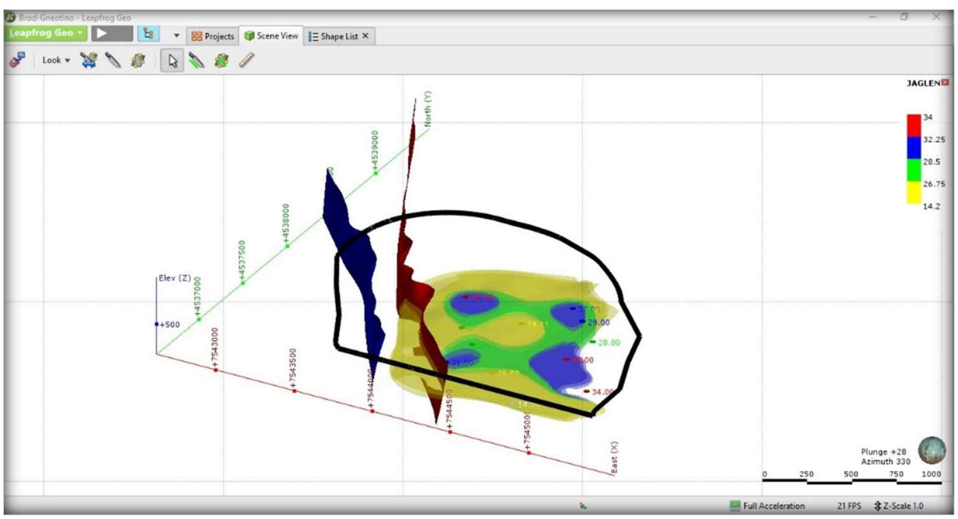

Figure 6: 3D representation of quasi-homogeneous domains of the angle of internal friction of the coal determined with the triaxial tests

The obtained value for the angle of the internal friction of the coal in the surface mine Brod-Gneotino is $\varphi = 24°$. From the 3D representation of the quasi-homogeneous domains of the angle of the internal friction of the coal, determined with triaxial tests, we can derive the following conclusions:

- ✓ Data used for creating of these domains are in the range from $\varphi = 14,20°$ to $\varphi = 34,00°$, with median $\varphi = 28,50°$ and are located in the non-fault domain,
- ✓ The domains with higher values ($28,50° - 32,25°$ and $32,25° - 34,00°$) are the least spatially represented and they are located mainly in the east final slopes, as well as in the line with the first fault structure in the surface mine,
- ✓ The domains with lower values ($14,20° - 26,75°$ and $26,75° - 28,50°$) are more spatially represented covering whole space in non-fault domain form the surface mine, especially the domain with the lowest values,
- ✓ It can be noticed that the obtained value for this geotechnical parameter belongs in the quasi-homogeneous domain with the lowest values which is spatially the most represented, but in stability analysis of the east final slopes, as well as in the line with the first fault structure, can be used higher values from the obtained ones.

3.5. Quasi-homogeneous domains of the angle of internal friction of coal determined with triaxial tests

The data used for modeling of the cohesion of the coal, determined by the triaxial tests belong in the geological parts of the five coal layers originated from Miocene period, hence the created quasi-homogeneous domains will refer to these parts from the geological model.

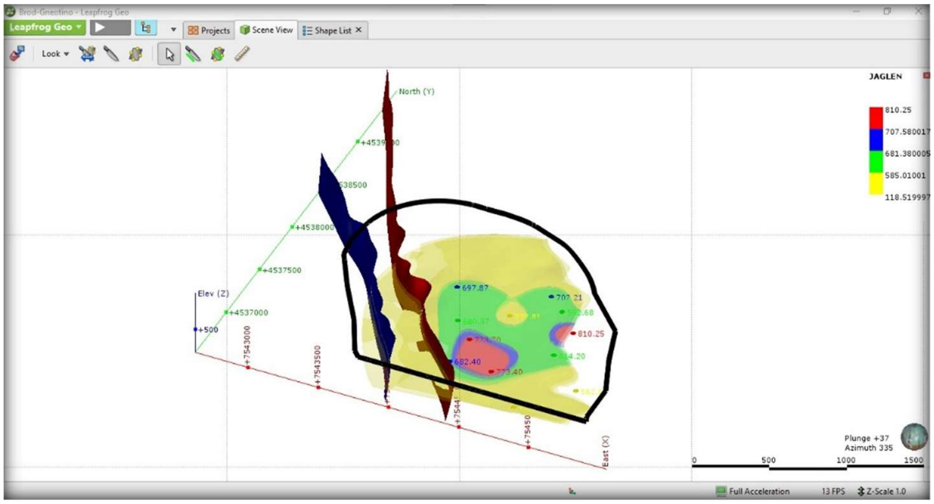

Figure 6: 3D representation of quasi-homogeneous domains of the cohesion of the coal determined with the triaxial tests

The obtained value for the cohesion of the coal in the surface mine Brod-Gneotino is $c = 50,00\ [kN/m^3]$. From the 3D representation of the quasi-homogeneous domains of the cohesion of the coal, determined with triaxial tests, we can derive the following conclusions:

- ✓ Data used for creating of these domains are in the range from $c = 118,52\ [kN/m^3]$ to $c = 810,25\ [kN/m^3]$, with median $c = 681,38\ [kN/m^3]$ and are located in the non-fault domain,
- ✓ The domains with higher values ($681,38 - 707,5\ [kN/m^3]$ and $707,58 - 810,25\ [kN/m^3]$) are the least spatially represented and they are located in the small part in the east final slopes, as well as in the middle part in the surface mine,
- ✓ The domains with lower values ($118,52 - 585,01\ [kN/m^3]$ and $585,01 - 681,38\ [kN/m^3]$) are more spatially represented covering whole space in non-fault domain form the surface mine.

CONCLUSIONS

The used mathematical models, aided by computer techniques, allow combination of geological model and numerical models for geotechnical parameters by geotechnical sections/domains, parts in combined models. These models display the distribution of values of the geotechnical parameters in different geological layers/mediums.

In the analysis of the stability of slopes in the excavation blocks at the surface mine Brod-Gneotino, the obtained values for geotechnical parameters are of primary importance. Considered quasi-homogeneous domains and their spatial analysis in the different parts of the surface mine, can make a significant contribution into adopting certain values for the geotechnical parameters, which on the other side are used for slope analysis. The scope of this paper, alongside with the theoretical approach of the theory of spatial analysis, especially semivariograms and kriging, is the selection of the geotechnical values needed for stability analysis of the final slopes of the surface mine. The studied domains show that in certain parts of the deposit, the obtained laboratory values are appropriate, while in other parts it can be used lower or higher values in order to be obtained more realistic analysis of the slope stability, as process, highly important for exploitation and long term planning and development of the open pit mine. Same approach can be suggested also for the case of underground mining.

All of this emphasize the importance of the spatial analysis aided with computer programs in the geotechnical modeling. The application of these methods is important for spatial and statistical processing of geotechnical data. Using corresponding interpolations results in selection of realistic values of the geotechnical parameters in all levels of project design is thus highly recommended.

This mathematical approach is quite general, easy for application and is of great benefit for geotechnical characterization of different types of mineral deposits.

Finally, due to the nature of the geological materials, other mathematical approaches can be suggested and are being used for geotechnical characterization in geotechnics such as in kinematic stability analyses for hard rock masses, calculations of bearing capacity, settlement, and others. All of these analysis find benefit from the geostatistical tools. These aspects overcome the scope of this paper.

COMPETING INTERESTS
Authors have declared that no competing interests exist.

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