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WORKING WITH MATHEMATICALLY GIFTED STUDENTS AGED 18 TO 19

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Abstract. The aims and objectives of all educational systems state that working with gifted students is of special social interest. At the same time, the care for these students comes down to organizing competitions and preparing the students for several days before the competitions. We believe that this approach does not even remotely meet the needs of gifted students, therefore, in this paper we have made an attempt to develop a curriculum for working with mathematically gifted students aged 18-19.

1. INTRODUCTION

Papers [1], [2] and [3] provide the curricula for working with mathematically gifted students from first, second and third year in the secondary education. This paper is actually a continuation of the abovementioned papers and it will provide an integral program for working with mathematically gifted students aged 18-19 years, that is, for students in the fourth year of secondary education. We deem that the preparation of such a curriculum that should necessarily be accompanied by appropriate books and collections of problems complementary to the curriculum will complete the work with mathematically gifted students in secondary education thus filling the existing gap in this field. In particular, such an approach will contribute to turn the declarative support of these students into real support, since the organization of competitions and the selective awarding of scholarships (scholarships are awarded to only a few of the best placed students in the competitions) is not enough to say that there is a serious social interest in the development of these children.

As we have already stated, this paper is in a way a continuation of the abovementioned papers. In addition, based on the experience of the authors, but also the experience of the countries in the immediate and wider surrounding, an attempt was made for part of the topic Generating functions to give an example of a system of problems that would determine the level that students should reach at this age.

2. CURRICULUM FOR WORKING WITH MATHEMATICALLY GIFTED STUDENTS AGED 18-19

In this section, we will present a curriculum for working with mathematically gifted students aged 18-19, that is, for students in the fourth year of secondary education. The offered curriculum actually builds on the respective teaching curriculum that was previously prepared for students in secondary education and is presented in papers [1], [2] and [3]. During the preparation of the curriculum, the method of concentric circles was used, which means that part of the contents that were adopted in the previous years at a certain level are expanded and extended. This curriculum should be implemented continuously, and not only in periods when students are preparing for certain math competitions.

The knowledge obtained while working with mathematically gifted students given in papers [4] and [6] was used during the development of this curriculum. The aims of the curriculum for students aged 18-19 are the following:

- To develop students' qualities of thinking such as: flexibility, stereotyping, width, rationality, depth and criticality,

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- The student to apply the scientific methods: observation, comparison, experiment, analysis, synthesis, classification, systematization and the axiomatic method,
- The student to apply the types of conclusions: induction, deduction and analogy, whereby it is of particular importance to present suitable examples from which the student will realize that the analogy conclusion is not always true,
- The student to adopt the prescribed contents in the field of functions of one real variable and to enable them to apply the same when solving appropriate problems,
- The student to adopt the prescribed contents in the field of differential and integral calculus of a function of one real variable and to be able to apply the acquired knowledge in problem solving,
- The student to adopt the prescribed contents in the field of inequalities and to be able to apply the acquired knowledge in problem solving,
- The student to adopt the prescribed contents in the field of combinatorics and to be able to apply the acquired knowledge in problem solving,
- The student to adopt the prescribed contents in the field of generating functions and to be able to apply the acquired knowledge in problem solving,
- The student to adopt the prescribed contents in the field of graph theory and to be able to apply the acquired knowledge in problem solving.

In order to achieve the aforementioned aims, it is necessary to adopt the following contents:

Analysis **(4 classes per week – 144 classes per year).** *Functions of one real variable*: basic properties of real functions, even, odd, periodic, monotone and bounded functions, elementary real functions (graphs), classification of real functions, parametrically defined functions and functions defined in polar coordinates, limit of a function at a point, the limits: $\lim_{x\to 0}\frac{\sin x}{x}$ and $\lim_{x\to\infty}(1+\frac{1}{x})^x$, continuous function at a point and of a set, elementary properties of continuous functions, properties of continuous functions on closed intervals.

Differential calculus of a function of one real variable: notion of derivative, basic properties and derivatives of elementary functions, derivative of inverse, composite and implicit function, higher order derivatives, calculating sums using derivatives, basic theorems of differential calculus (Fermat, Rolle, Lagrange and Cauchy), L'Hôpital's rule, equation of a tangent, angle between two curves, Maclaurin and Taylor formula, monotonicity and local extrema of a function, application, convexity and concavity of a function, points of inflection, asymptotes of a curve, construction of a function graph.

Integral calculus of a function of one variable: notion of primitive function and indefinite integral, change of variables, partial integration, integration of rational functions, the notion of definite integral, basic properties of a definite integral, connection between definite and indefinite integral, change of variables and partial integration for a definite integral, area of a plane figure, arc length of a plane curve.

Inequalities: proving inequalities using monotonicity of a function, proving inequalities using extrema of a function, the inequalities of Popoviciu, Jensen, Bernoulli, Young, Jordan, Hölder, Minkowski, Karamata, Schur, Muirhead, Petrovic, Nesbitt, Hadwiger-Finsler, weighted inequalities of the means, inequalities of the means of order s and order r, notion of symmetric inequality, symmetric inequalities with three variables, normalization procedure and application of differential calculus in proving symmetric inequalities.

Selected contents from discrete mathematics **(3 classes per week – 108 classes per year).** *Combinatorics:* partition of a number, ordered partition of a number, partition of a set, derangements, games and strategies, problems with coloring, covering and dissecting, weight and acquaintance problems, double counting and Hall's theorem.

Generating functions: concept of generating function, operations with generating functions, generating functions and differential equations, Hadamard product for rational generating functions, application of generating functions in the theory of enumeration, generating functions and partitions, exponential generating functions, harmonic numbers, sums of powers of natural numbers, Bernoulli polynomials and Bernoulli numbers, Catalan numbers, the snake oil method.

Graph theory: notion of graph, isomorphic graphs, matrix representation of graphs, types of graphs, subgraphs, degree of a vertex, regular graph, graph operations, trails and cycles, connectivity, trees, cyclomatic number of a graph, cutset, cuts, properties of adjacency and incidence matrices, Eulerian and Hamiltonian graphs, planar graphs and characterization of planar graphs. Matching in graphs. The notion of matching. System of distinct representatives. Perfect matching theorem. Coloring graphs. Chromatic number of a graph. Graphs with large chromatic number.

3. EXAMPLE OF SYSTEM OF PROBLEMS FOR SECTION "GENERATING FUNCTIONS"

In order to realize the suggested curriculum for working with gifted 18-19 years old students, it is necessary to make appropriate teaching aids, that is to say, textbooks that must be accompanies by appropriate books with collections of problems. Hereinafter, we will present a system of problems that we deem is suitable for studying the section Generating functions and which tasks are selected from the books [5], [7] and [8].

1. Determine the generating function of:

   a) binomial coefficient of $n=$th order $\binom{n}{k}$, $k=0,1,2,...,n$,

   b) sequence $a_k=(-1)^k$, $k=0,1,2,3,...$

2. Let $g_1(x)$ be the generating function of the sequence $\{a_i\}_{i=0}^{\infty}$ and $g_2(x)$ be the generating function of the sequence $\{b_i\}_{i=0}^{\infty}$. Prove that $g(x)=g_1(x)+g_2(x)$ is a generating function of the sequence $\{c_i\}_{i=0}^{\infty}$, where $c_i=a_i+b_i$, $i=0,1,2,3,...$.
3. Let $g(x)$ be the generating function of the sequence $\{a_i\}_{i=0}^{\infty}$ and c be a constant. Prove that $f(x)=cg(x)$ is the generating function of the sequence $\{ca_i\}_{i=0}^{\infty}$.
4. Let $g_1(x)$ be the generating function of the sequence $\{a_i\}_{i=0}^{\infty}$, $g_2(x)$ be the generating function of the sequence $\{b_i\}_{i=0}^{\infty}$ and $\alpha,\beta\in\mathbb{R}$. Prove that $g(x)=\alpha g_1(x)+\beta g_2(x)$ is a generating function of the sequence $\{c_i\}_{i=0}^{\infty}$, where $c_i=\alpha a_i+\beta b_i$, $i=0,1,2,3,...$.
5. Let $g(x)$ be the generating function of the sequence $\{a_i\}_{i=0}^{\infty}$. Prove that $x^n g(x)$ is a generating function of the sequence $\{b_i\}_{i=0}^{\infty}$, where $b_k=0, k=0,1,2,...,n-1$ and $b_k=a_{k-n}, k\geq n$.
6. Determine the generating function of the sequence $a_k=1$, $k=0,1,2,3,...$.
7. Let $g(x)$ be the generating function of the sequence $\{a_i\}_{i=0}^{\infty}$. Prove that

$$\frac{g(x)-a_0-a_1x-a_2x^2-...-a_{n-1}x^{n-1}}{x^n}$$

is the generating function of the sequence $a_n, a_{n+1}, a_{n+2}, \ldots$.

8. Let $g(x)$ be the generating function of the sequence $\{a_i\}_{i=0}^{\infty}$. Prove that $g(cx)$ id the generating function of the sequence $\{c^i a_i\}_{i=0}^{\infty}$.

9. a) Let $g_1(x)$ be the generating function of the sequence $\{a_i\}_{i=0}^{\infty}$ and $g_2(x)$ be the generating function of the sequence $\{b_i\}_{i=0}^{\infty}$. Prove that $g(x) = g_1(x)g_2(x)$ is the generating function of the sequence $\{c_i\}_{i=0}^{\infty}$ where

$$c_n = \sum_{k=0}^{n} a_k b_{n-k} = a_0 b_n + a_1 b_{n-1} + a_2 b_{n-2} + \ldots + a_{n-1} b_1 + a_n b_0,$$

b) Let $g_1(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$ be the generating function such that $g_1(0) = a_0 \neq 0$. Prove that there exists a generating function

$$g_2(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + \ldots$$

such that $g_1(x)g_2(x) = 1$. Hence we write $g_2(x) = \frac{1}{g_1(x)}$.

10. For the sequence $\{a_n\}_{n=0}^{\infty}$ determine the generating function of the partial sums of the series $\sum_{i=0}^{\infty} a_i$.

11. If $g(x)$ is a generating function for the sequence $\{a_n\}_{n=0}^{\infty}$, then $g'(x)$ is the generating function for the sequence $\{na_n\}_{n=1}^{\infty}$. Prove it!

12. If $g(x)$ is a generating function for the sequence $\{a_n\}_{n=0}^{\infty}$, then $\int_0^x g(t)dt$ is the generating function for the sequence $0, a_0, \frac{a_1}{2}, \frac{a_2}{3}, \ldots$. Prove it!

13. Determine the generating functions of the sequences $\{n+1\}_{n=0}^{\infty}$ and $0, 1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots$.

14. The numbers $H_0 = 0$ and $H_k = 1 + \frac{1}{2} + \ldots + \frac{1}{k}$, for $k = 1, 2, \ldots$ are called *harmonic numbers* and they are equal to the partial sums of the harmonic series $\sum_{k=0}^{\infty} \frac{1}{k+1}$.

a) Prove that for each natural number m, the following is valid $H_{2^m} \geq 1 + \frac{m}{2}$.

b) Determine the generating function for the harmonic series.

15. Prove that for each natural number m the following is valid $H_{2^m} \leq 1 + m$.

16. Prove that for $n \geq 2$ the following is valid $\sum_{k=2}^{n} \frac{1}{k(k-1)} H_k = 2 - \frac{H_{n+1}}{n} - \frac{1}{n+1}$.

17. Determine the generating function of the sequence $a_n = \frac{1}{(n+1)(n+2)}$, $n = 0,1,2,\dots$.

18. Prove that for every $m \geq 1$ the following is valid

$$\frac{1}{(1-ax)^m} = 1 + \binom{m}{1} ax + \binom{m+1}{2} a^2 x^2 + \binom{m+2}{3} a^3 x^3 + \dots + \binom{m+n-1}{n} a^n x^n + \dots$$

19. Determine the generating function for the sequence:

a) $a_n = \frac{1}{(n+5)!}$, $n = 0,1,2,\dots$,

b) $a_n = \frac{n^2+n+1}{n!}$, $n = 0,1,2,\dots$,

20. Determine the generating function for the number b_n of integers in the interval from 0 to $10^m - 1$ whose sum of digits is equal to n.

21. Solve the differential equation $a_0 = 5$, $a_k = a_{k-1} + 3$, за $k \geq 1$.

22. Using generating functions derive the formula for the general term of the Fibonacci sequence: $f_0 = 0$, $f_1 = 1$, $f_{n+2} = f_{n+1} + f_n$, for $n \geq 0$.

23. Let the sequence $\{a_n\}_{n=0}^{\infty}$ satisfy the linear differential equation

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_m a_{n-m}$$

from $m-$th order with constant coefficients $c_1, c_2, \dots, c_m$ and starting conditions $a_0, a_1, \dots, a_m$.

Prove that the generating function $g(x)$ of this sequence is of the kind $g(x) = \frac{P_{m-1}(x)}{Q_m(x)}$, where $Q_m(x)$ is a polynomial of $m-$th degree, and the degree of the polynomial $P_{m-1}(x)$ is smaller or equal to $m-1$.

24. Let the generating function

$$g(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_{m-1} x^{m-1} + a_m x^m + \dots \quad (1)$$

be rational, that is, if $g(x) = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are coprime polynomials. Prove that, starting from a number n the sequence $\{a_n\}_{n=0}^{\infty}$ satisfies the linear differential equation

$$a_{n+m} = c_1 a_{n+m-1} + c_2 a_{n+m-2} + \dots + c_m a_n, \quad (2)$$

where m is the degree of the polynomial $Q(x)$, and $c_1, c_2, \dots, c_m$ are some constants.

25. Solve the differential equation $a_0 = 1$, $a_1 = 4$, $a_k = a_{k-1} + 6a_{k-2}$, $k \geq 2$.

26. The sequence $\{a_n\}_{n=0}^{\infty}$ is given with a recurrence relation $a_0 = 2, a_1 = 7,\ a_{n+2} = 4a_{n+1} - 4a_n + 3^n,\ n \geq 0$. Determine the explicit formula for a_n.

27. Solve the differential equation $a_0 = 1,\quad a_k = 3a_{k-1} + 4^k, k \geq 1$

28. Solve the differential equation $a_0 = 3,\quad a_k = 2a_{k-1} + k, k \geq 1$.

29. For every natural number n, let

$$S_n = 1 + \tfrac{1}{2} + \ldots + \tfrac{1}{n},$$
$$T_n = S_1 + S_2 + \ldots + S_n,$$
$$U_n = \tfrac{T_1}{2} + \tfrac{T_2}{3} + \ldots + \tfrac{T_n}{n+1}.$$

For every n determine the constants a_n, b_n, c_n, d_n such that

$$T_n = a_n S_{n+1} + b_n \text{ and } U_n = c_n S_{n+1} + d_n.$$

30. Prove that the generating function for the sequence $\{a_i\}_{i=0}^{\infty}$ is rational if and only if there exist numbers $q_1, q_2, \ldots, q_k$ and polynomials $p_1(t), p_2(t), \ldots, p_k(t)$ such that starting from some n the following is valid

$$a_n = p_1(t)q_1^n + p_2(t)q_2^n + \ldots + p_k(t)q_k^n. \qquad (1)$$

The expression on the right-hand side from (1) is called *quasipolynomial* from the variable n.

31. *Hadamard product* for the generating functions

$$g(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots \text{ and } h(x) = b_0 + b_1x + b_2x^2 + b_3x^3 + \ldots$$

is called the generating function

$$f(x) = a_0b_0 + a_1b_1x + a_2b_2x^2 + a_3b_3x^3 + \ldots.$$

Prove that the Hadamard product for two rational generating functions is a rational generating function.

32. Let $g(s) = \frac{P_1(s)}{Q_1(s)}$ and $h(s) = \frac{P_2(s)}{Q_2(s)}$ be rational generating functions, given with coprime fractions and let $(g * h)(s) = \frac{P(s)}{Q(s)}$ be their Hadamard product, written as a fraction in simplest form. What can be said about the polynomial $Q(s)$, if we know the polynomials $Q_1(s)$ and $Q_2(s)$?

33. Determine the number of solutions of the equation $e_a + e_b + e_c + e_d = r$, where $0 \leq r \leq 6$, $e_a, e_b, e_c, e_d \in \mathbb{Z}$ and $0 \leq e_a \leq 1, 0 \leq e_b \leq 1, 0 \leq e_c \leq 2, 0 \leq e_d \leq 2$.

34. Let A be a set that contains 1 object of type a, 1 object of type b, 2 objects of type c and 2 objects of type d. Determine the number of ways in which we can choose 4 objects from the given 4 types in the set A.

35. A box contains 4 red, 5 blue and 2 green balls.

a) In how many different ways can 7 balls be chosen from the box?

b) In how many different ways can 7 balls be chosen but there must be 1 red and 2 blue balls?

36. Let us assume that there are 3 red, 8 green, 9 orange and 2 white balls in a box. In how many ways can we choose 12 balls if we have to choose at least one red ball, an even number of green balls and an odd number of orange balls?

37. Determine the coefficient in front of x^{24} in the series $(x^3+x^4+x^5+x^6+...)^4$.

38. In how many ways can 12 objects be chosen from 5 types of objects, if there are at most 2 objects from the first three types, and unlimited number of objects from the remaining two types?

39. In how many ways can 20 objects be selected, if objects from the first type can only be selected in packages of 5, of the second type only in packages of 3 objects each, of the third type can only be selected 4 at most, of the fourth type at least 3 objects and at most 2 objects from the fifth type?

40. Find the generating function whose $n-$th coefficient gives the number of non-negative solutions of the equation $e_1+4e_2+5e_3+3e_4=n$.

41. Prove that the function $\frac{1}{(1-x)(1-x^2)(1-x^3)...(1-x^k)...}$ is a generating function whose $n-$th coefficient gives the number of ways of placing of n same objects into n same boxes, so that some boxes can remain empty, that is, gives the number of partitions of the number n to n or less partitions (non-negative integer summands).

42. Using generating functions determine the number of ways to partition the number n as the sum of n or fewer distinct numbers.

43. Prove that the number of ways to partition the natural number n, as the sum of n or fewer distinct natural numbers is equal to the number of ways to partition the natural number n as the sum of n or fewer odd natural numbers.

44. a) For the natural number n let f_n be the number of subsets of the set $\{1,2,...,n\}$ which do not contain a pair of consecutive numbers. Determine the recurrence (differential) equation that these numbers satisfy, and then find the numbers.

b) For the natural numbers n and k let $f_{n,k}$ be the number of $k-$subsets of the set $\{1,2,...,n\}$ that do not contain a pair of consecutive numbers. Determine the recurrence equation that these number satisfy, and then determine the appropriate generating function and the numbers.

45. The function $h(x)=\sum\limits_{k=0}^{\infty}\frac{a_k}{k!}x^k$ is called *exponential generating function* for the sequence $\{a_k\}_{k=0}^{\infty}$. Prove that if f and g are exponential generating function for the sequences $\{a_k\}_{k=0}^{\infty}$ and $\{b_k\}_{k=0}^{\infty}$, then
a) the function $f(x)+g(x)$ is exponential generating function for the sequence $\{a_k+b_k\}_{k=0}^{\infty}$.
b) the function $f(x)g(x)$ is exponential generating function for the sequence

$$c_n=\sum_{k=0}^{n}\binom{n}{k}a_k b_{n-k},\ n=0,1,2,3,\dots,$$

which is called *binomial convulsion of the sequences* $\{a_k\}_{k=0}^{\infty}$ and $\{b_k\}_{k=0}^{\infty}$.
46. Determine the exponential generating function of the sequence $a_k=\frac{n!}{(n-k)!},\ k=0,1,2,\dots,n$.
47. Let us assume that we have unlimited number of white, black, green and blue balls. Determine the number of ways in which we can choose and rearrange 2 white, 4 black, 3 green and 3 blue balls.
48. Determine the exponential generating function that can be used to find the number of ways in which n persons can be accommodated into 3 rooms with at least 2 but not more than 9 persons.
49. Determine the exponential generating function for the sequence of variations with repetition from n elements from k -th class.
50. Determine the number of accommodating n guests in three halls, so that in the first hall there must be at least one guest, in the second hall there must be an odd number of guests, and in the third hall there must be an even number of guests.
51. Using the exponential generating functions prove that the number of ways in which n distinct objects can be put in k different boxes and no box remains empty and и $1\le k\le n$ is $A_k^{(n)}=\sum\limits_{i=0}^{k}(-1)^i\binom{k}{i}(k-i)^n$.
52. Prove that the number of ways in which n distinct objects can be put in k same boxes so that no box remains empty and $1\le k\le n$ is

$$S_k^{(n)}=\frac{1}{k!}\sum_{i=0}^{k}(-1)^i\binom{k}{i}(k-i)^n,$$

where $S_k^{(n)}$, $0\le k\le n$ are Stirling numbers of the second kind.

53. $2n$ points are given on a circle. In how many ways can these points be partitioned into n pairs so that between these $n-$chords determined by these pairs of points there are no two that intersect?

54. For the string of zeros and ones with length $2n$ ($2n$-string over alphabet $\{0,1\}$) we shall say it is *balanced* if it contains n zeros and n ones. For the balanced $2n$-string over alphabet $\{0,1\}$ we shall say it is good if none of its initial parts have more zeros than ones. Otherwise we will say that $2n$-string is bad.

Prove that the number of good $2n$-strings over alphabet $\{0,1\}$ is $C_n = \frac{1}{n+1}\binom{2n}{n}$.

55. There are $2n$ persons standing in queue in front of the ticket office. Each of them wants to buy a ticket that costs 50 denars. Among the people in the queue, exactly n persons have 50 denars, whereas the rest have one banknote of 100 denars. At the beginning, the cash desk is empty. What is the number of customer arrangements so that the salesperson can return the change to every person buying a ticket?

56. Find the number of sequences with length $2n$: $a_1, a_2, ..., a_{2n}$ with elements from the set $\{-1,1\}$ such that $\sum_{k=1}^{2n} a_k = 0$ and $\sum_{k=1}^{m} a_k \geq 0$ for $1 \leq m < 2n$.

57. Find the number of sequences with length n whose elements are integers $a_1, a_2, ..., a_n$ such that $1 \leq a_1 \leq a_2 \leq ... \leq a_n$ and $a_1 \leq 1, a_2 \leq 2, ..., a_n \leq n$.

58. The rook has to pass from the lower left to the upper right corner square of a chessboard with dimensions $n \times n$. The rook can only move from left to right and from bottom to top. How many different paths are there to achieve the goal if at no time can the rook be placed above the diagonal that connects the starting and ending square?

59. *An anti-Pascal* triangle is an equilateral triangular array of numbers such that, except for the numbers in the bottom row, each number is the absolute value of the difference of the two numbers immediately below it. For example, the following is an anti-Pascal triangle with four rows which contains every integer from 1 to 10.

4

2 6

5 7 1

8 3 10 9

Is there an anti-Pascal triangle with 2018 rows which contains every integer from 1 to $1+2+...+2018$?

60. Using the Snake oil method, prove that:

a) $\sum_k \binom{n}{k}\binom{k}{j} x^k = \binom{n}{j} x^j (1+x)^{n-j}$, for each $n \geq 0$, б) $\sum_{k}\binom{2n+1}{2k}\binom{m+k}{2n}=\binom{2m+1}{2n}$,

в) $\sum_{k=0}^{n}\binom{2n}{2k}\binom{2k}{k}2^{2n-2k}=\binom{4n}{2n}$

4. CONCLUSION

Previously, we provided a curriculum for working with gifted students aged 18-19 and for one of the topics contained in it we presented a system of problems that we believe is sufficient to achieve the aims of the curriculum from the given topic. Among other things, we believe that the acquisition of the theoretical knowledge envisaged by this curriculum, supported by appropriate collections of problems, will enable:

- Formation of students' qualities of thinking at an enviable level,
- Students to apply the types of inferences correctly: induction, deduction and analogy,
- Students apply the scientific methods correctly: observation, comparison, experiment, analysis, synthesis, classification, systematization and the axiomatic method, and
- Students to acquire the necessary knowledge needed for their future development, that is, for the successful continuation of their academic career at the best universities.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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