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STATISTICAL ANALYSIS OF REPAIR RATE FOR MAINTENANCE DECISION-MAKING

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Abstract The most important part of any maintenance process is to achieve the optimal level of system availability. Having reviewed the available literature, we noticed that the availability mostly depends on the number of spare parts in stock, reliability, and repair time. Due to that fact, in this paper, we are analyzing the stochastic nature of the repair process. The aim is to determine the repair time for the required level of availability. First, we analyze the repair time of a single component. The final equation of the mathematical model that we present herein is the probability density function of the repair rate which allows us to determine the repair time for the related level of availability and mean time to failure. Further, based on this equation, we present the approach for the determination of the repair time of a series system comprised of two components. The model's output can be used in making important decisions such as the planning of maintenance activities, capacity, labor planning, etc.

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1. INTRODUCTION

Maintenance comprises a set of procedures and methods for keeping the system in an operational state or returning the system to that state after failure [1]. So, when the component or system fails, it needs to be repaired or replaced. Each of these activities requires a certain time for their implementation. This period is usually called downtime. Since many factors influence the delay duration, they can be divided into waiting and active downtimes. Waiting downtimes are delays that occur due to waiting for spare parts, administrative procedures, deliveries, staff, etc. Active downtimes are used to repair or replace a component or system, so the repair time can be observed as a random variable. In this paper, we will observe only repairable systems i.e. systems that can be returned to their functional state with certain activities, after the occurrence of failure.

The key performance measures of both repairable and non-repairable systems are availability and reliability. Availability is defined as the probability that a system will perform its function in a time [2]. Conversely, reliability is defined as the probability that the equipment or the system will complete a specific task under specified conditions for a stated time [3]. The main goal of each maintenance system (MRO) is to achieve the desired level of availability and for that purpose, maintenance contracts have been used. Their characteristic is that no specific maintenance activities such as servicing, repairs, and required materials are paid for, but only performances of the system resulting from the undertaken maintenance activities. This concept originates from the military industry i.e. it is related to the maintenance of military aircraft and weapon systems. Maintenance contracts have also found their use in civilian companies, under the name Performance Based Contracts (PBC) [4]. In practice, it comes down to this, when the component under a PBC contract is serviced, maintenance is not charged by the number of working hours used for engine repair or by the number of used spare parts, but by the time in which the airplane is available after repairs i.e. number of hours the engine is in an operational state. [5]

Kang et al. [6] have observed systems whose maintenance was regulated with maintenance contracts. They concluded that the mean time to failure (MTTF), mean time to repair (MTTR), and the number of spare parts impact availability. Evaluating the availability of a certain component or system is a common topic in related literature. Inherited availability and methods for its evaluation in repairable systems have been researched in papers [7 - 9]. Papers [10-13] contributed significantly to determining the availability of repairable systems and components under maintenance contracts. A similar issue was researched in the paper [14] and it was concluded that repair time and reliability have a significantly greater effect on the system's availability than the number of spare parts in inventory.

Based on the previous, this paper presents the model for the repair rate of one component and the series system comprised of two components.

2. MODELING OF REPAIR RATE

Stochastic modeling of the component or system repair time is not new and has been already presented in the paper [15]. The method presented in [16] observes the repair rate as a stochastic process and aims to determine this parameter for the preferred level of unit availability. Only repairable components and systems were considered, actually the systems that alternate between successive up and down intervals. We assumed that at the start system is operative. It remains in that state for a certain time T (failure time), then it stops operating for time R (repair time) and after being repaired system is back in the operative state. This process of failure and repair will repeat. In the literature, this approach is known as the alternating renewal process [17] and it is defined as a sequence of independent and non-negative random variables. In this case, the random variables are the times-to-failure and the times-to-repair/restore. Each time a component fails and is restored to working order, a renewal occurred.

We also assumed that perfect repair has been carried out at a constant rate after which the system behaves the same as the new one. As already stated, the main purpose of maintenance contracts is to optimize system availability and in the case when we have a maintenance contract, inherited or steady state availability is often used availability measure. The steady-state availability is inherited availability when considering only the corrective downtime of the system and is equal to:

$$A(\infty) = \lim_{t \to \infty} A(t) \tag{1}$$

According to the key renewal theorem the limited probability that the system is available can be expressed as the ratio of the mean of the period when the system is operative and the mean of the period which represents one renewal cycle [18]:

$$A = \lim_{t \to \infty} A(t) = \frac{E[T]}{E[T] + E[R]} = \frac{MTTF}{MTTF + MTTR}, \tag{2}$$

where E is the expected value operator. Further, $MTTF$ is a random variable and if there exists probability density function $p(t)$, then the $MTTF$ can be defined as:

$$MTTF = \int_0^\infty t p(t) dt. \tag{3}$$

We assumed that failure time has Rayleigh distribution so the probability density function is:

$$p(t) = {}^{2t}\!/_{x} \exp\left(-{}^{t^2}\!/_{x}\right) \tag{4}$$

where the distribution parameter x is determined by relation $E(t^2) = x$.
By replacing (4) with (3) the equation for $MTTF$ is:

$$MTTF = \int_0^\infty \frac{2t^2}{x} \exp\left(-\frac{t^2}{x}\right) dt. \tag{5}$$

From the maintenance theory, we know that the rate of repair μ can be observed as a reciprocal value of $MTTR$ as in Eq. (6)

$$\mu = 1 / MTTR\,. \tag{6}$$

Based on the all presented facts in the paper [16] and according to the previous, the probability density function (PDF) of the repair rate can be stated as:

$$p(\mu) = \frac{8A^2}{\left(1-A\right)^2 \mu^3 \pi x_0} \exp\left(\frac{-4A^2}{(1-\mathrm{A})^2 \mu^2 \pi x_0} \right), \tag{7}$$

while the CDF can be calculated as:

$$F(\mu) = \int_0^{\mu_r} p(\mu)\,\mathrm{d}\,\mu = 1 - \exp\left(\frac{-4A^2}{\left(1-\mathrm{A}^2\right)\mu^2 \pi x_0} \right). \tag{8}$$

Further, we will observe the repair rate of the series system of two components. If one component fails, the system will fail too. This system can be illustrated in Fig. 1

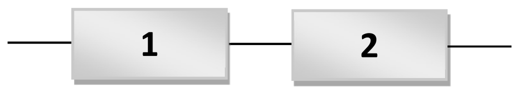

Fig1. The series system of two components

The repair rate of such a system can be expressed as $\mu_s = \mu_1 \cdot \mu_2$, so the PDF function of the system repair rate is:

$$\mathrm{p}(\mu_s) = \int_0^{+\infty} \frac{1}{\mu_2} \mathrm{p}_1\left(\frac{\mu_s}{\mu_2}\right) \mathrm{p}_2\left(\mu_2\right) \mathrm{d}\mu_2 . \tag{9}$$

Since the repair rate of part 1 is $\mu_1 = \dfrac{\mu_s}{\mu_2}$ then the PDF function of the first part is:

$$p\left(\frac{\mu_s}{\mu_2}\right) = \frac{8A_1^2}{\left(1-A_1\right)^2 \left(\frac{\mu_s}{\mu_2}\right)^3 \pi x_{01}} \exp\left(\frac{-4A_1^2}{(1-A_1)^2 \left(\frac{\mu_s}{\mu_2}\right)^2 \pi x_{01}} \right). \tag{10}$$

Similarly, the PDF of the second part is:

$$p(\mu_2) = \frac{8A_2{}^2}{\left(1-A_2\right)^2 \mu_2^3 \pi x_{02}} \exp\left(\frac{-4A_2^2}{(1-A_2)^2 \mu_2^2 \pi x_{02}} \right). \tag{11}$$

Now, we can determine the PDF of the series system repair rate as:

$$p\left(\mu_s\right) = \frac{64 A_1 A_2}{\left(1\text{-}A_1^2\right)\left(1\text{-}A_2^2\right)\pi x_{01} x_{02} \mu_s^3} \int_0^{+\infty} \frac{1}{\mu_2} \exp\left(\frac{-4A_1^2 \mu_2}{\left(1\text{-}A_1^2\right)\mu_s^2 \pi x_{01}} + \frac{-4A_2^2}{\left(1\text{-}A_2^2\right)\mu_2^2 \pi x_{02}} \right) d\mu_2 . \tag{12}$$

By solving the integral in the previous in Eq., we get the following:

$$p(\mu_s) = \frac{64 A_1 A_2}{\left(1 - A_1^2\right)\left(1 - A_2^2\right)\pi x_{01} x_{02} \mu_s^3} K_0\left(\frac{8 A_1 A_2}{\mu_s \pi \sqrt{\left(1 - A_1^2\right)\left(1 - A_2^2\right) x_{01} x_{02}}}\right), \tag{13}$$

where K_0 is BesselK function [19].

NUMERICAL RESULTS

In this section, the model presented in the previous section will be tested for two components. We assumed the failure rates λ of both components are known ($\lambda_1 = 1.5$ and $\lambda_2 = 2$).

Numerical analysis was conducted to calculate the annual expected time for the repair to acquire availability of A=0.85, A=0.9, A=0.95 by emphasizing the stochastic nature of this process. A similar analysis can also be conducted for other values of parameter A.

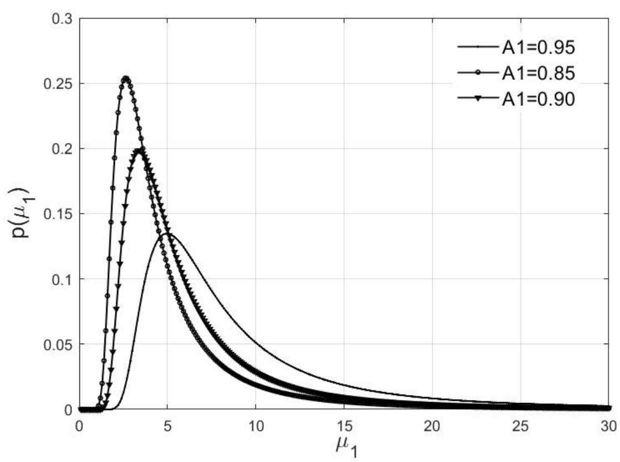

Fig 2. PDF function of the first component (λ=1.5)

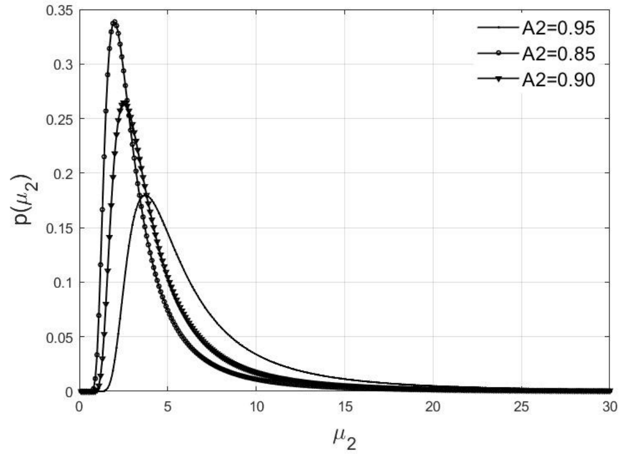

Fig 3. PDF function of the second component (λ=2)

Fig.2 represents the probability of repair rate of the first part depending on time for cases when it is expected that availability of this component is 85%, 90%, and 95%. Likewise, Fig.3 provides data related to the second part.

Finally, we have determined the repair rate of the series system comprised of these two parts based on Eq. (13). The results have been illustrated in Fig4.

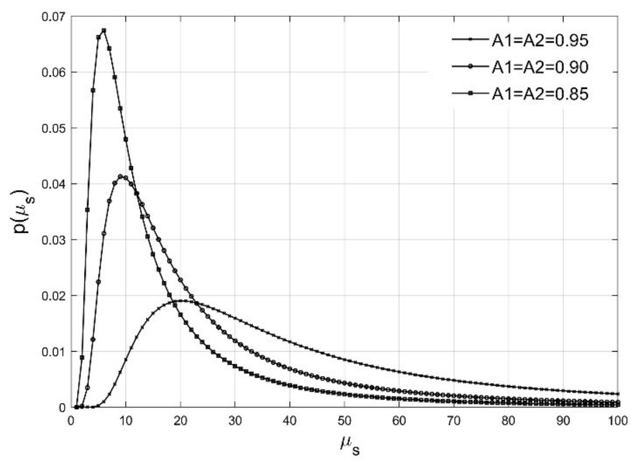

Fig 4. The repair rate of the series system comprised two components

The time necessary to repair such a system, as can be seen in Fig. 4, is known to be longer than the time necessary to repair individual parts.

5. CONCLUSION

Related research on repairable systems' maintenance processes showed that reliability and repair rate have a significant impact on the availability of such systems. In this paper, we examined a repairable system that can be modeled with an alternating renewal process. First, we observed separate units/components of such a system. We assumed that the failure rate is Rayleigh distributed and that the *MTTF* is a predetermined value. Also, after repairs, the unit returned to its original state and performed as new. By observing repair time as a stochastic process, we presented the exact expressions for the repair rate's PDF. After determining the repair rate characteristics of a single unit or subsystem, we calculated the PDF of a series system of two components. In the Numerical section, the proposed model was applied to the system comprised of two components. The PDF of the repair rate for each component was graphically presented as well as the PDF of the series system. Based on this information we can conclude in which time interval maintenance action should be completed to achieve the desired level of availability. Even though we set availability on certain levels, the numerical analysis can be repeated with different values of availability. This model can be applied in the same manner to other repairable systems with the alternating renewal process. The obtained results can be used in the planning of maintenance activities, inventory, service systems, and the number of required employees, in the process of system maintenance.

COMPETING INTERESTS

The authors have declared that no competing interests exist.

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