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N-TUPLE WEAK ORBITS TENDING TO INFINITY FOR HILBERT SPACE OPERATORS

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Abstract . In this paper we prove some results on the existence of a dense set of pairs in the direct product of an infinite-dimensional complex Hilbert space with itself such that each pair in this set has an *n*-tuple weak orbit tending to infinity for a specific countable family of mutually commuting bounded linear operators.

1. INTRODUCTION

For bounded linear operators on Banach spaces the concepts of *n*-tuple orbits and *n*-tuple weak orbits are defined as follows. If X is a complex and infinite-dimensional Banach space, $B(X)$ is the algebra of all bounded linear operators on X and $T_1, T_2, \ldots, T_n \in B(X)$ are mutually commuting operators, then the *n-tuple orbit* of the vector $x \in X$ is the set

$$\operatorname{Orb}(\{T_i\}_{i=1}^{n}, x) = \left\{ T_1^{k_1} T_2^{k_2} \ldots T_n^{k_n} x : k_i \geq 0; 1 \leq i \leq n \right\}. \tag{1.1}$$

The *n*-tuple orbit *tends to infinity* if

$$\lim_{k_i \to \infty} \left\| T_1^{k_1} T_2^{k_2} \ldots T_n^{k_n} x \right\| = \infty, \text{ for all } k_j \geq 0,\ j \neq i,\ 1 \leq i, j \leq n .$$

For $n = 1$, the *n*-tuple orbit (1.1) reduces to a simple sequence of form

$$\operatorname{Orb}(T, x) = \left\{ T^n x : n = 0, 1, 2, \ldots \right\},$$

usually referred as *single orbit* (or simply *orbit*) of the vector $x \in X$ under the operator T . If X^* is the dual space of X , i.e., the space of all bounded linear functionals $x^* : X \to \mathbb{C}$, and for $x \in X$ and $x^* \in X^*$, $\langle x, x^* \rangle := x^*(x)$, the *n-tuple weak orbit* of the pair $(x, x^*) \in X \times X^*$ is a set of form

$$\operatorname{Orb}(\{T_i\}_{i=1}^{n}, x, x^*) = \left\{ \left\langle T_1^{k_1} T_2^{k_2} \ldots T_n^{k_n} x, x^* \right\rangle : k_i \geq 0; 1 \leq i \leq n \right\}. \tag{1.2}$$

The *n*-tuple weak orbit *tends to infinity* if

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$$\lim_{k_i \to \infty} \left| \left\langle T_1^{k_1} T_2^{k_2} \dots T_n^{k_n} x, x^* \right\rangle \right| = \infty \text{, for all } k_j \geq 0 \text{, } j \neq i \text{, } 1 \leq i, j \leq n \text{ .}$$

For $n = 1$, the *n*-tuple weak orbit (1.2) reduces to a simple scalar sequence of form

$$\mathrm{Orb}(T, x, x^*) = \left\{ \left\langle T^n x, x^* \right\rangle : n = 0,1,2,\dots \right\},$$

usually referred as *weak orbit* of the pair $(x, x^*) \in X \times X^*$ under the operator T .

For the case of Hilbert spaces, by the Riesz Theorem for representation of a bounded linear functional on Hilbert spaces (cf. [7,III.6]), given an infinite-dimensional complex Hilbert space H with an inner product $\langle \cdot | \cdot \rangle$, its dual space H^* can be fully identified with the space itself since

$$H^* = \left\{ x \mapsto \langle x | y \rangle, x \in H : y \in H \right\} .$$

Hence, for a set of mutually commuting operators $T_1, T_2, \dots, T_n \in B(H)$ the *n*-tuple weak orbits will be the sets of form

$$\mathrm{Orb}(\{T_i\}_{i=1}^n, x, y) = \left\{ \left\langle T_1^{k_1} T_2^{k_2} \dots T_n^{k_n} x \middle| y \right\rangle : k_i \geq 0; 1 \leq i \leq n \right\} \text{, } (x, y) \in H \times H \text{ .}$$

In this paper we will consider only the conditions under which the direct product $H \times H$ contains a dense of pairs (x, y) with *n*-tuple weak orbits tending to infinity that do not involve any requirements upon specific subsets of the spectra of the operators. For $H \times H$ we will assume that is endowed with the product topology. Given an operator $T \in B(H)$, $\sigma(T)$ and $r(T)$ will denote the spectrum and the spectral radius of the operator T , respectively.

2. PRELIMINARY RESULTS

Theorem 2.1. ([6, Theorem V.39.8]) *Let* H *and* K *be Hilbert spaces,* $(T_n)_{n \geq 1}$ *be a sequence of operators in* $B(H, K)$ *and* $(a_n)_{n \geq 1}$ *be sequence of positive numbers with* $\sum_{n \geq 1} a_n < \infty$ *. Then*

(i) *there are* $x \in H$ *and* $y \in K$ *such that and* $\left| \langle T_n x | y \rangle \right| \geq a_n \| T_n \|$ *, for all* n ;

(ii) *there is a dense subset of pairs* $(x, y) \in H \times K$ *such that* $\left| \langle T_n x | y \rangle \right| \geq a_n \| T_n \|$, *for all* n *sufficiently large.*

Corollary 2.2. ([6, Corollary V.39.9]) *Let* H *be Hilbert space and* $T \in B(H)$ *is such that* $\sum_{k=1}^{\infty} \left\| T^k \right\|^{-1} < \infty$ *. Then there exist* $x, y \in H$ *such that* $\left| \langle T^n x | y \rangle \right| \to \infty$ *. Moreover, the set of such pairs* (x, y) *is dense in* $H \times H$ *.*

Lemma 2.3. ([6, Lemma V.37.15]) *Let* $\varepsilon > 0$ *and* $(a_n)_{n\geq 1}$ *be a sequence of positive numbers satisfying* $\sum_{n\geq 1} a_n < \varepsilon$. *Then there is a sequence of positive numbers* $(b_n)_{n\geq 1}$ *such that* $b_n \to \infty$ *as* $n \to \infty$ *and* $\sum_{n\geq 1} a_n b_n < \varepsilon$.

3. Main Results

Let $F = \{1, 2, \ldots, N\}$ for some $N \in \mathbb{N}$, $N \geq 2$, or $F = \mathbb{N}$.

Theorem 3.1. *Let* H *be a Hilbert space,* $\{T_i : i \in F\} \subset B(H)$ *and* $\{(a_{i,j})_{j\geq 1} : i \in F\}$ *be a family of sequences of positive numbers such that* $\sum_{i\in F, j\geq 1} a_{i,j} < \infty$. *Then for any open balls* B_1 *and* B_2 *in* H *there are vectors* $x \in B_1$, $y \in B_2$ *and* $k_0 \in \mathbb{N}$ *such that*

$$\left|\left\langle T_i^k x \middle| y \right\rangle\right| \geq a_{i,k} \left\| T_i^k \right\|, \textit{ for all } i \in F \textit{ and } k \geq k_0.$$

Proof. Let $T_{i,k} := T_i^k$ ($i \in F$, $k \in \mathbb{N}$), $f : F \times \mathbb{N} \to \mathbb{N}$ be the bijective mapping defined with

$$f(i,j) = \begin{cases} i + N(j-1), & \text{if } F = \{1,2,\ldots,N\} \\ \dfrac{(i+j-2)(i+j-1)}{2} + j, & \text{if } F = \mathbb{N} \end{cases},$$

and let $g : \mathbb{N} \to F \times \mathbb{N}$ denote its inverse mapping. If $(a'_n)_{n\geq 1}$ is a sequence of positive numbers and $(T'_n)_{n\geq 1}$ is a sequence of operators defined with

$$a'_n = a_{g(n)} \text{ and } T'_n = T_{g(n)}, \text{ for all } n \in \mathbb{N},$$

then $\sum_{n\geq 1} a'_n = \sum_{i\in F, j\geq 1} a_{i,j} < \infty$. Hence (by Theorem 2.1. (ii), applied on $(a'_n)_{n\geq 1}$, $(T'_n)_{n\geq 1}$ and $H = K$), if B_1 and B_2 are open balls H, then there are $x \in B_1$, $y \in B_2$ and $n_0 \in \mathbb{N}$ such that

$$\left|\left\langle T'_n x \middle| y \right\rangle\right| \geq a'_n \left\| T'_n \right\|, \text{ for all } n \geq n_0. \tag{3.1}$$

Since $f : F \times \mathbb{N} \to \mathbb{N}$ is bijective, there is a unique pair $(i_0, j_0) \in F \times \mathbb{N}$ such that $n_0 = f(i_0, j_0)$. Let

$$k_0 = \begin{cases} j_0 + 1, & \text{if } F = \{1,2,\ldots,N\} \\ i_0 + j_0, & \text{if } F = \mathbb{N} \end{cases}.$$

If $(i,k) \in F \times \mathbb{N}$ is such that $k \geq k_0$, then by the definition of $f : F \times \mathbb{N} \to \mathbb{N}$ we have:

1. for $F = \{1, 2, \ldots, N\}$,

$$f(i,k) = i + N(k-1) \geq N(k_0 - 1) = Nj_0 = N + N(j_0 - 1)$$
$$\geq i_0 + N(j_0 - 1) = n_0 ,$$

2. for $F = \mathbb{N}$,

$$f(i,k) = \frac{(i+k-2)(i+k-1)}{2} + k \geq \frac{(i_0 + j_0 - 2)(i_0 + j_0 - 1)}{2} + j_0 = n_0.$$

Hence, by (3.1) and the definition of $(a'_n)_{n\geq 1}$ and $(T'_n)_{n\geq 1}$ we obtain

$$\left|\left\langle T_i^k x \middle| y \right\rangle\right| = \left|\left\langle T_{i,k} x \middle| y \right\rangle\right| = \left|\left\langle T_{g(n)} x \middle| y \right\rangle\right| \geq a_{g(n)} \left\| T_{g(n)} \right\| = a_{i,k} \left\| T_i^k \right\|,$$

for all $i \in F$ and $k \geq k_0$. ■

Theorem 3.2. *If H is Hilbert space and $\{T_i : i \in F\} \subset B(H)$ is a family of operators such that $\sum_{k=1}^{\infty} \left\| T_i^k \right\|^{-1} < \infty$, for all $i \in F$, then there is a dense set $D \subset H \times H$ such that the weak orbit $\left(\left\langle T_i^k x \middle| y \right\rangle\right)_{k\geq 0}$ tends to infinity for every pair $(x, y) \in D$ and every $i \in F$. If, in addition, $\{T_i : i \in F\}$ is a family of mutually commuting operators such that the sequence $(T_i^k - T_j^k)_{k\geq 1}$ is norm bounded for all $i, j \in F$, then for every $n \in F$ and $1 < m \leq n$, the $m-$tuple weak orbit*

$$\left\{\left\langle T_{i_1}^{k_1} T_{i_2}^{k_2} \dots T_{i_m}^{k_m} x \middle| y \right\rangle : k_i \geq 0; 1 \leq i \leq m\right\},$$

tends to infinity for all $1 \leq i_1 < i_2 < \dots < i_m \leq n$.

Proof. Let B_1 and B_2 be open balls H. For $i \in F$, let $\varepsilon_i > 0$ be such that

$$\varepsilon_i \left(\sum_{k=1}^{\infty} \frac{1}{\left\| T_i^k \right\|} \right) < \frac{1}{2^{i+1}},$$

and (by Lemma 2.3) let $(b_{i,k})_{k\geq 1}$ be the sequence of positive numbers such that $b_{i,k} \to \infty$ as $k \to \infty$ and

$$\sum_{k=1}^{\infty} \frac{\varepsilon_i b_{i,k}}{\left\| T_i^k \right\|} < \frac{1}{2^{i+1}}. \tag{3.2}$$

If $a_{i,k} = \varepsilon_i b_{i,k} \left\| T_i^k \right\|^{-1}$, $(i,k) \in F \times \mathbb{N}$, then by (3.2) we have

$$\sum_{i\in F, k\geq 1} a_{i,k} = \sum_{i\in F} \sum_{k=1}^{\infty} \frac{\varepsilon_i b_{i,k}}{\left\| T_i^k \right\|} < \sum_{i\in F} \frac{1}{2^{i+1}} < \frac{1}{2}.$$

Hence, by Theorem 3.1, there are $x \in B_1$, $y \in B_2$ and $k_0 \in \mathbb{N}$ such that

$$\left|\left\langle T_i^k x \middle| y\right\rangle\right| \geq a_{i,k}\left\|T_i^k\right\| = \varepsilon_i b_{i,k}\left\|T_i^k\right\|^{-1}\left\|T_i^k\right\| = \varepsilon_i b_{i,k}, \text{ for all } i \in F \text{ and } k \geq k_0 .$$

Letting $k \to \infty$, we have

$$\lim_{k\to\infty}\left|\left\langle T_i^k x \middle| y\right\rangle\right| = \infty, \text{ for all } i \in F . \tag{3.3}$$

If, in addition, $\{T_i : i \in F\}$ is a family of mutually commuting operators such that the sequence $(T_i^k - T_j^k)_{k\geq 1}$ is norm bounded for all $i, j \in F$, let $M_{i,j} > 0$ is such that $\left\|T_i^k - T_j^k\right\| \leq M_{i,j}$, for all $k \geq 0$, and let $(x, y) \in H \times H$ be a pair satisfying (3.3). We continue by induction.

Let $m = 2$ and $1 \leq i_1 < i_2 \leq n$. By the Cauchy-Bunyakovsky-Schwarz inequality we have

$$\begin{aligned}\left|\left\langle T_{i_1}^{k_1+k_2} x \middle| y\right\rangle\right| &\leq \left|\left\langle T_{i_1}^{k_1+k_2} x - T_{i_1}^{k_1}T_{i_2}^{k_2} x \middle| y\right\rangle\right| + \left|\left\langle T_{i_1}^{k_1}T_{i_2}^{k_2} x \middle| y\right\rangle\right| \\ &= \left|\left\langle T_{i_1}^{k_1}(T_{i_1}^{k_2} - T_{i_2}^{k_2})x \middle| y\right\rangle\right| + \left|\left\langle T_{i_1}^{k_1}T_{i_2}^{k_2} x \middle| y\right\rangle\right| \\ &\leq \left\|T_{i_1}^{k_1}(T_{i_1}^{k_2} - T_{i_2}^{k_2})x\right\| \cdot \|y\| + \left|\left\langle T_{i_1}^{k_1}T_{i_2}^{k_2} x \middle| y\right\rangle\right| \\ &\leq \left\|T_{i_1}^{k_1}\right\| \cdot \left\|T_{i_1}^{k_2} - T_{i_2}^{k_2}\right\| \cdot \|x\| \cdot \|y\| + \left|\left\langle T_{i_1}^{k_1}T_{i_2}^{k_2} x \middle| y\right\rangle\right| \\ &\leq \left\|T_{i_1}\right\|^{k_1} \cdot M_{i_1,i_2} \cdot \|x\| \cdot \|y\| + \left|\left\langle T_{i_1}^{k_1}T_{i_2}^{k_2} x \middle| y\right\rangle\right| .\end{aligned}$$

Since $\left|\left\langle T_{i_1}^n x \middle| y\right\rangle\right| \to \infty$ as $n \to \infty$ (hence $\left|\left\langle T_{i_1}^{k_1+k_2} x \middle| y\right\rangle\right| \to \infty$ as $k_2 \to \infty$, for all $k_1 \geq 0$), the above inequalities imply that

$$\left|\left\langle T_{i_1}^{k_1}T_{i_2}^{k_2} x \middle| y\right\rangle\right| \to \infty, \text{ as } k_2 \to \infty, \text{ for all } k_1 \geq 0 .$$

Similarly,

$$\begin{aligned}\left|\left\langle T_{i_2}^{k_1+k_2} x \middle| y\right\rangle\right| &\leq \left|\left\langle T_{i_2}^{k_1+k_2} x - T_{i_1}^{k_1}T_{i_2}^{k_2} x \middle| y\right\rangle\right| + \left|\left\langle T_{i_1}^{k_1}T_{i_2}^{k_2} x \middle| y\right\rangle\right| \\ &= \left|\left\langle T_{i_2}^{k_2}(T_{i_2}^{k_1} - T_{i_1}^{k_1})x \middle| y\right\rangle\right| + \left|\left\langle T_{i_1}^{k_1}T_{i_2}^{k_2} x \middle| y\right\rangle\right| \\ &\leq \left\|T_{i_2}\right\|^{k_2} \cdot M_{i_2,i_1} \cdot \|x\| \cdot \|y\| + \left|\left\langle T_{i_1}^{k_1}T_{i_2}^{k_2} x \middle| y\right\rangle\right| ,\end{aligned}$$

which implies that

$$\left|\left\langle T_{i_1}^{k_1}T_{i_2}^{k_2} x \middle| y\right\rangle\right| \to \infty, \text{ as } k_1 \to \infty, \text{ for all } k_2 \geq 0 .$$

To complete the proof, it is enough to show the claim is true for $m = n$, under the assumption that the (n-1)-tuple weak orbit

$$\left\{\left\langle T_{i_1}^{k_1} T_{i_2}^{k_2} \dots T_{i_{n-1}}^{k_{n-1}} x \middle| y \right\rangle : k_j \ge 0; 1 \le j \le n-1 \right\},$$

tends to infinity for all $1 \le i_1 < \dots < i_{n-1} \le n$. For a fixed $i \in \{1,2,\dots,n\}$, arbitrary $j \in \{1,2,\dots,n\} \setminus \{i\}$ and fixed $k_1, k_2, \dots, k_n \ge 0$ we have

$$\begin{aligned}
&\left|\left\langle T_1^{k_1} \dots T_{i-1}^{k_{i-1}} T_j^{k_i} T_{i+1}^{k_{i+1}} \dots T_n^{k_n} x \middle| y \right\rangle\right| \\
&\le \left|\left\langle T_1^{k_1} \dots T_{i-1}^{k_{i-1}} T_j^{k_i} T_{i+1}^{k_{i+1}} \dots T_n^{k_n} x - T_1^{k_1} T_2^{k_2} \dots T_n^{k_n} x \middle| y \right\rangle\right| + \left|\left\langle T_1^{k_1} T_2^{k_2} \dots T_n^{k_n} x \middle| y \right\rangle\right| \\
&= \left|\left\langle T_1^{k_1} \dots T_{i-1}^{k_{i-1}} T_{i+1}^{k_{i+1}} \dots T_n^{k_n} (T_j^{k_i} - T_i^{k_i}) x \middle| y \right\rangle\right| + \left|\left\langle T_1^{k_1} T_2^{k_2} \dots T_n^{k_n} x \middle| y \right\rangle\right| \\
&\le \left\| T_1^{k_1} \dots T_{i-1}^{k_{i-1}} T_{i+1}^{k_{i+1}} \dots T_n^{k_n} (T_j^{k_i} - T_i^{k_i}) x \right\| \cdot \|y\| + \left|\left\langle T_1^{k_1} T_2^{k_2} \dots T_n^{k_n} x \middle| y \right\rangle\right| \\
&\le \left(\prod_{\substack{l=1 \\ l \ne i}}^{n} \|T_l\|^{k_l} \right) \cdot M_{i,j} \cdot \|x\| \cdot \|y\| + \left|\left\langle T_1^{k_1} T_2^{k_2} \dots T_n^{k_n} x \middle| y \right\rangle\right|.
\end{aligned}$$

Since $j \in \{1,2,\dots,n\} \setminus \{i\}$, by the inductive assumption, we have

$$\left|\left\langle T_1^{k_1} \dots T_{i-1}^{k_{i-1}} T_j^{k_i} T_{i+1}^{k_{i+1}} \dots T_n^{k_n} x \middle| y \right\rangle\right| \to \infty \text{ as } k_i \to \infty, \text{ for all } k_j \ge 0,\ j \ne i.$$

This, together with the above inequalities implies that

$$\left|\left\langle T_1^{k_1} T_2^{k_2} \dots T_n^{k_n} x \middle| y \right\rangle\right| \to \infty \text{ as } k_i \to \infty, \text{ for all } k_j \ge 0,\ j \ne i,$$

which completes the proof. ■

Corollary 3.3. *If H is a Hilbert space and $\{T_i : i \in F\} \subset B(H)$ is a family of operators such that $r(T_i) > 1$, for all $i \in F$, then there is a dense set $D \subset H \times H$ such that the weak orbit $\left(\left\langle T_i^k x \middle| y \right\rangle\right)_{k \ge 0}$ tends to infinity for every pair $(x, y) \in D$ and every $i \in F$. If, in addition, $\{T_i : i \in F\}$ is a family of mutually commuting operators such that the sequence $(T_i^k - T_j^k)_{k \ge 1}$ is norm bounded for all $i, j \in F$, then for every $n \in F$, every $1 < m \le n$ the $m-$tuple weak orbit*

$$\left\{\left\langle T_{i_1}^{k_1} T_{i_2}^{k_2} \dots T_{i_m}^{k_m} x \middle| y \right\rangle : k_i \ge 0; 1 \le i \le m \right\},$$

tends to infinity for all $1 \le i_1 < i_2 < \dots < i_m \le n$.

Proof. If $T \in B(H)$ has a spectral radius $r(T) > 1$, then $\sum_{k=1}^{\infty} \left\|T^k\right\|^{-1} < \infty$. Namely, if $r(T) > 1$, then there is $\lambda \in \sigma(T)$ such that $1 < |\lambda|$. By the Spectral Mapping Theorem, $\lambda^n \in \sigma(T^n)$ for every $n \in \mathbb{N}$. Hence $|\lambda|^n \le r(T^n) \le \|T^n\|$ and

$$\sum_{n=1}^{\infty} \frac{1}{\|T^n\|} \leq \sum_{n=1}^{\infty} \frac{1}{|\lambda|^n} < \infty .$$

Now the conclusion follows from Theorem 3.2. ■

4. REMARKS ON N-TUPLE ORBITS TENDING TO INFINITY

By the Cauchy-Bunyakovsky-Schwarz inequality we have

$$\left|\left\langle T_1^{k_1} T_2^{k_2} \ldots T_n^{k_n} x \middle| y \right\rangle\right| \leq \left\|T_1^{k_1} T_2^{k_2} \ldots T_n^{k_n} x\right\| \cdot \|y\|,$$

for all $(x, y) \in H \times H$, $k_j \geq 0$ and $1 \leq j \leq n$. These inequalities clearly imply that the *n*-tuple orbit $\mathrm{Orb}(\{T_i\}_{i=1}^n, x)$ tends to infinity whenever there is $y \in H$ such that the *n*-tuple weak orbit $\left\{\left\langle T_1^{k_1} T_2^{k_2} \ldots T_n^{k_n} x \middle| y \right\rangle : k_i \geq 0; 1 \leq i \leq n\right\}$ tends to infinity. Hence, from the results in the previous section we can derive the following results for *n*-tuple orbits tending to infinity.

Theorem 4.1. *If* H *is Hilbert space and* $\{T_i : i \in F\} \subset B(H)$ *is a family of operators such that* $\sum_{k=1}^{\infty} \left\|T_i^k\right\|^{-1} < \infty$ *for all* $i \in F$ *, then there is a dense set* $D \subset H$ *such that the orbit* $\mathrm{Orb}(T_i, x)$ *tends to infinity for every* $x \in D$ *and every* $i \in F$ *. If, in addition,* $\{T_i : i \in F\}$ *is a family of mutually commuting operators such that the sequence* $(T_i^k - T_j^k)_{k \geq 1}$ *is norm bounded for all* $i, j \in F$ *, then for every* $n \in F$ *, every* $1 < m \leq n$ *, the* $m-$*tuple orbit* $\mathrm{Orb}(\{T_{i_j}\}_{j=1}^m, x)$ *tends to infinity for all* $1 \leq i_1 < i_2 < \ldots < i_m \leq n$ *.*

Corollary 4.2. *If* H *is Hilbert space and* $\{T_i : i \in F\} \subset B(H)$ *is a family of operators such that* $r(T_i) > 1$ *for all* $i \in F$ *, then there is a dense set* $D \subset H$ *such that the orbit* $\mathrm{Orb}(T_i, x)$ *tends to infinity or every* $x \in D$ *and every* $i \in F$ *. If, in addition,* $\{T_i : i \in F\}$ *is a family of mutually commuting operators such that the sequence* $(T_i^k - T_j^k)_{k \geq 1}$ *is norm bounded for all* $i, j \in F$ *, then for every* $n \in F$ *, every* $1 < m \leq n$ *, the* $m-$*tuple orbit* $\mathrm{Orb}(\{T_{i_j}\}_{j=1}^m, x)$ *tends to infinity for all* $1 \leq i_1 < i_2 < \ldots < i_m \leq n$ *.*

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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