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About the geometric interprations of the basic interactions and some consequences

Kostadin Trenčevski
Ss. Cyril and Methodius University in Skopje, Arhimedova 3, Skopje, Macedonia, e-mail: kostadin.trencevski@gmail.com

Abstract

Our space-time consists of three 3-dimensional spaces: space S, space rotations SR and time T. The basic spaces are S and SR, while in case of constraints of these two spaces, the body will be time displaced, of change the speed of time as in case of gravitational field. In recent papers [15, 14] it is given a (possible) geometric description of basic interactions in the nature, by using the non-commutativity of translations and rotations in two groups of rotations and translations together: in the affine group, and the group which is locally isomorphic to $SO(4)$, or $Spin(4)$. In this paper our attention is mainly to the classical electromagnetic interactions and gravitational interactions. The electromagnetic interaction can be interpreted such that both charged particles are mutually rotated for an angle, while the gravitational interaction can be interpreted such that the gravitational body with mass M radially translates each point for length GM/c^2. Using these two interpretations, in this paper we prove that the mass is observed enlarged for coefficient $1/\sqrt{1-\frac{v^2}{c^2}}$, while the charge is observed unchanged according to the observer who moves with velocity $\mathbf{v}$. These two results are ad hoc used in physics, but now we have a deduction of them.

1 Introduction

The space and time were subject of interest from the old civilizations up to the present time. They were separated many centuries ago. Even in the Newton theory they are still separated and the time flow was considered as uniform phenomena in the universe. Remarkable approach in understanding the space and time was done by the well known scientist and philosopher Roger Boscovich (1711-1787), who was not well understood at that time. He made distinction between the real space-time and the space-time according to our observations (ref. [1]). More than one century before the Special Relativity, he wrote that there does not exist an absolute space in rest, i.e. about the relativity among the moving systems.

According to the Theory of Relativity there does not exist strong separation between the space and time, which is evident from the Lorentz transformations. This idea was generalized in the recent refs. ([2, 3, 4]) for the space, time and rotations. For each small body besides its 3 spatial coordinates, can be jointed also 3 degrees of freedom about its rotation in the space and also 3 degrees of freedom for the velocity of the considered body. These 3+3 degrees of freedom are of the same level and importance as the basic 3 spatial coordinates. There are three 3-dimensional sets: space S which is homeomorphic to S^3, spatial rotations SR which is also homeomorphic to S^3 and time T which is homeomorphic to $\mathbb{R}^3$. The space SR is homeomorphic to S^3 if it is considered as the group of quaternions with module 1, which is locally isomorphic to $SO(3,\mathbb{R})$. Each two of these sets may interfere analogously to the space and time in the Special Relativity.

The group of Lorentz transformations $O_+^\uparrow(1,3)$ is isomorphic to $SO(3,\mathbb{C})$, and if we consider this complex group as a group of real 6×6 matrices, this group is the required Lie group which connects the spaces S and T. The group which connects the spaces SR and T is the same group of transformations. This Lie group of transformations has Lie algebra which is determined by the matrices of type

$$\begin{bmatrix} X & Y \\ -Y & X \end{bmatrix}, \tag{1}$$

where X and Y are antisymmetric 3×3 matrices. This Lie group will be denoted by G_{t}, because it connects the space T (temporal space) with the other two spaces. In ref. [4] the Lorentz transformations are converted as transformations in $S\times T$, given by 6×6 matrices.

The Lie group which connects the spaces S and SR has Lie algebra which consists of matrices of type

$$\begin{bmatrix} X & Y \\ Y & X \end{bmatrix}, \tag{2}$$

where X and Y are antisymmetric 3×3 matrices. This Lie group is generated by the following 3 matrices of translation $\tau_{(\alpha,0,0)}$, $\tau_{(0,\alpha,0)}$ and $\tau_{(0,0,\alpha)}$ along the x, y, and z axes, where

$$\tau_{(\alpha,0,0)} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos\alpha & 0 & 0 & 0 & \sin\alpha \\ 0 & 0 & \cos\alpha & 0 & -\sin\alpha & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \sin\alpha & 0 & \cos\alpha & 0 \\ 0 & -\sin\alpha & 0 & 0 & 0 & \cos\alpha \end{bmatrix},$$

while the other two matrices $\tau_{(0,\alpha,0)}$ and $\tau_{(0,0,\alpha)}$ are obtained by the cyclic permutation $(1,2,3,4,5,6)\mapsto(2,3,1,5,6,1)$ and also by the following 3 matrices of rotation $\rho_{(\alpha,0,0)}$, $\rho_{(0,\alpha,0)}$ and $\rho_{(0,0,\alpha)}$ around the x, y, and z axes, where

$$\rho_{(\alpha,0,0)} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha & 0 & 0 & 0 \\ 0 & -\sin\alpha & \cos\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\alpha & \sin\alpha \\ 0 & 0 & 0 & 0 & -\sin\alpha & \cos\alpha \end{bmatrix},$$

while the other two matrices $\rho_{(0,\alpha,0)}$ and $\rho_{(0,0,\alpha)}$ are obtained by the same cyclic permutation. This group will be denoted by G_s as a space group, which connects the spaces S and SR. The group G_s is isomorphic to the group Spin(4) ([5]). We denote by $\mathcal{A}$ the affine group of translations and rotations in the Euclidean space, and as a set of 6×6 matrices it can be proved that its Lie algebra has the form

$$\begin{bmatrix} X & Y \\ 0 & X \end{bmatrix}, \qquad (3)$$

where X and Y are antisymmetric 3×3 matrices.

The elements of the space S are measured in meters, while the elements of spatial rotations SR are measured in radians, and so there exists a local constant as a coefficient of proportionality between these two spaces, which is called *radius of range* R. The elementary particles, galaxies and the universe, have their own radii of range. While the velocity of light c connects the space and time, the radius of range connects the space and space rotations.

The multi-dimensional time was investigated also by another authors, for example in refs. [6, 7, 8, 9, 10, 11, 12, 13].

2 Exchanging among S, SR and T

The are 4 basic exchanges among the spaces S, SR and T. If their elements are denoted by s, r and t respectively, then the basic 4 exchanges are [14, 15]:

$$1.\ r \to s, \quad 2.\ s \to r, \quad 3.\ r \to t, \quad 4.\ s \to t.$$

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Figure 1: Basic four exchanges among the spaces S, SR and T.

In general $x \mapsto y$ means that if $x \in X$ is constrained to occur, then it will be converted into $y \in Y$. According to 1. and 2., when one of them is constrained then it converts into the other space from $S \times SR$. The case 3. says that when the rotation r is constrained, then it makes some changes in the time, for example in speed of time. But, if the rotation r is constrained, then it tries first to convert into $s \in S$, and if it is not admitted, then it converts into $t \in T$. So, the case 3.

must be of composite type $s \to r \to t$. Analogously, the case 4. must be of type $r \to s \to t$. But, the cases $t \mapsto s$ and $t \mapsto r$ are not admitted, because none may constrain the time. Also it is clear that the composite cases $s \to r \to s$ are not admitted. We give some simple examples given in [14, 15].

The first case $r \to s$ means that when the rotation is not permitted, then the particles will be translated, i.e. will be displaced. It occurs for example, when the spinning bodies are moving in a circle, or a spinning football ball moves on an arc in the air, instead of parabolic trajectory. In both cases, each point of the spinning body intends to rotate according to its trajectory in the space. But it is not completely admitted, because the body is solid. As a consequence, each point of the spinning body intends to be displaced (or translated) in the space and it moves according to the sum of all such small displacements. In general some of these displacements are also constrained, and in such a motion we obtain also changes of type 4. This is commented in many details in [16]. This displacement in the previous papers were called induced spin motions, or simply spin motions. These spin motions are non-inertial motions. For example, if a football ball stops to rotate around its non-constant axis, then it will continue to move according to the well known parabolic trajectory. Moreover, in [16] it is explained the variation of the length of the day with a period of 6 months. It is important to mention that if the spin motion is constrained, then it becomes inertial motion.

The second case $s \to r$ means that if the space displacement is not admitted completely or partially, then it induces a spatial rotation. For example, let a rigid body moves with a velocity $\mathbf{v}$. If one point S of the rigid body is constrained to move, then the body will start to rotate around the point S. So the rotation is induced in this case. Analogously to the spin motion, in this case we also have both cases 2. and 3.

Assume that a non-rotating body initially rests with respect to the Earth on almost infinity distance. Assume that this body freely falls toward the Earth under the Earth's gravitation. When the body comes at the surface of the Earth, it is not permitted to be displaced further. So, this constraint will cause time displacement, such that the time will be slower. Indeed, if the velocity at the surface is equal to v, then the constraint for the space displacement will induce slower time for coefficient $\lambda = \sqrt{1 - \frac{v^2}{c^2}}$. This is the case 4 ($s \to t$). Since $v \approx \sqrt{2GM/R}$, the time on the surface of the Earth is slower for coefficient $\lambda \approx \sqrt{1 - \frac{2GM}{Rc^2}} \approx 1 - \frac{GM}{Rc^2}$, which is also well known from the General Relativity up to approximation of c^{-2}. This example is important to mention in order to emphasize that the speed of time in gravitational field is slower because of the existence of the acceleration toward the center of the planet, but not conversely. Indeed, the gravitational acceleration is not a consequence since the speed of time is not constant close to the planet.

It is also interesting to mention if someone intends to construct a time-travel machine, it is necessary to use the cases 3. and 4.

3 The induced 4 cases and the basic interactions in the space.

Each of these four cases induces an interaction in the space. In general, it leads to global classification of the basic interactions in the nature. Let O_1 and O_2 be the centers of the bodies and X is close to O_2 which belongs to the second body, such that $\overrightarrow{O_2X} = (a, b, c)$ (Fig. 2), and let τ be a translation for arbitrary small vector $\overrightarrow{O_2X} = (a, b, c)$. The operator rotor below should be done with respect to the coordinates a, b and c. We consider a solid body, such that the vector τ can not be constrained. Hence we have the following 4 cases (see also Table 1):

1^*. The electro-weak interaction is a consequence of non-commutativity between τ and one rotation in the space group G_s. The rotation is partially constrained in G_{s}, and as a consequence it appears an induced translation in G_{s}, which occurs as electro-weak interaction.

2^*. The strong interaction is a consequence of non-commutativity between τ and one translation in the space group G_s. The translation is partially constrained in G_{s} and as a consequence it appears an induced angle or rotation, which leads to displacement observed as acceleration.

3^*. The electromagnetic interaction is a consequence of non-commutativity between τ and one rotation in the affine group $\mathcal{A}$. The rotation is partially constrained in $\mathcal{A}$, and as a consequence it appears an induced translation in the affine group $\mathcal{A}$, and further it leads to electromagnetic interaction.

4^*. The gravitational interaction is a consequence of non-commutativity between τ and one "radial translation" in the affine group $\mathcal{A}$. The translation is partially constrained in $\mathcal{A}$, and as a consequence it appears an induced angle of rotation, which leads further to gravitational interaction.

Group of trans.	rotation	translation
G_{s}	electro-weak int.	strong int. & gravity-weak int.
$\mathcal{A}$	electromagnetic int.	gravitational interaction

Table 1: Global scheme of the basic interactions.

In all interactions, beside the acceleration $\mathbf{a}$ it appears also an angular velocity $\mathbf{w}$. In case of the electromagnetic and gravitational interaction these two 3-dimensional vectors are parts of an antisymmetric tensor field among the inertial systems, which is analogous to the tensor of the electromagnetic field. So, these two interactions are called *temporal interactions.* In case of weak and strong interactions, we have again two vector fields $\mathbf{a}$ and $v_0\mathbf{w}$ instead of $c\mathbf{w}$, where v_0 is a local parameter analogous to the radius of range R. Indeed, the velocity c is characteristic only for electromagnetic and gravitational interactions. Instead of an antisymmetric tensor, in case of weak and strong interaction we have the following theorem ([15]):

Theorem 1. *In case of weak and strong interaction, i.e. in the Lie group* G_{s}*, it holds* $\mathbf{a} = v_0\mathbf{w}$ *or* $\mathbf{a} = -v_0\mathbf{w}$*.*

So, these two interactions (weak and strong) are called *spatial interactions*.

We will give a short presentations of the interactions classified in the following way: i) strong interaction, ii) electromagnetic and electro-weak interaction, and iii) gravitational and gravity-weak interaction. The following results were obtained in [15] and some of them also in [14].

3.1 Strong interaction

We need to present the observation from the center of arbitrary particle with radius of range R. In [15] is shown that in case of the polar coordinates only the distance r is changed and it is observed as $R\sin\frac{r}{R}$. Moreover, the metric is given by

$$(ds)^2=\Big(\cos\frac{r}{R}\Big)^2(dr)^2+\Big(\frac{R}{r}\sin\frac{r}{R}\Big)^2r^2[(d\phi)^2+\sin^2\phi(d\theta)^2]=$$

$$=R^2\Big[\Big(d(\sin\frac{r}{R})\Big)^2+\Big(\sin\frac{r}{R}\Big)^2((d\phi)^2+\sin^2\phi(d\theta)^2)\Big]. \tag{4}$$

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Figure 2: The strong interaction is a consequence of non-commutativity of translations for the vectors $\mathbf{r}$ and (a,b,c) in $S\times SR$.

Let us consider two nucleons with centers at O_1 and O_2 with radii of range R_1 and R_2 respectively, and $\mathbf{r}=\overrightarrow{O_1O_2}$ (Fig.2). The non-commutativity of the translations obtains by the angle of two translations: translation for vector $\overrightarrow{O_1X}$ observed by O_1, and then translation by vector $-\mathbf{r}$ observed by the point X, or almost the same by O_2. In G_{s} the angle as a result of two translations is analogous to rotation as a consequence of two rotations, i.e. the vector product of two vectors and we use the coefficient 1/2 analogous to the Thomas precession. The endpoint of translation is a point Y which is close to O_1, where almost there is no rotation between O_1 and Y. We use the notations $k_1=\frac{R_1}{r}\sin\frac{r}{R_1}$, $k_2=\frac{R_2}{r}\sin\frac{r}{R_2}$, and $k_1^*=\cos\frac{r}{R_1}$. Without loss of generality we assume that the vector $\mathbf{r}$ is parallel to the z-axis, i.e. $x=y=0$, and as a consequence of the metric, the vector $\overrightarrow{O_1X}$ is observed from O_1 as $k_1\mathbf{r}+(k_1a,k_1b,k_1^*c)$, while the vector $-\mathbf{r}$ from X is observed as $-k_2\mathbf{r}$. The normalization should be done with respect to the distance r. Using the form of matrices given in section 1, the angle φ is given by

$$\vec{\varphi}=\frac{-1}{2}\Big[\frac{k_1\mathbf{r}+(k_1a,k_1b,k_1^*c)}{r}\times\frac{-k_2\mathbf{r}}{r}\Big]=-k_1k_2\frac{\mathbf{r}\times(a,b,c)}{2r^2}.$$

Further we obtain

$$\mathrm{rot}\vec{\varphi}=-k_1k_2\frac{\mathbf{r}}{r^2},$$

$$\frac{1}{2}\mathrm{rot}\vec{\varphi}=-\frac{1}{2}\sin\frac{r}{R_1}\sin\frac{r}{R_2}\frac{R_1R_2}{r^2}\frac{\mathbf{r}}{r^2}. \tag{5}$$

Assume that the rotation is not admitted. The relative acceleration between the two bodies is given by

$$\mathbf{a}_{\rm rel} = -\frac{v_0^2}{2} \sin\frac{r}{R_1} \sin\frac{r}{R_2} \frac{R_1 R_2}{r^2} \frac{\mathbf{r}}{r^2}. \tag{6}$$

Let us denote by $\mathbf{a}_1$ and $\mathbf{a}_2$ the accelerations of the first and the second body, then

$$\mathbf{a}_{\rm rel} = \mathbf{a}_2 - \mathbf{a}_1, \quad \mathbf{a}_1 = -\frac{m_2}{m_1+m_2}\mathbf{a}_{\rm rel}, \quad \mathbf{a}_2 = \frac{m_1}{m_1+m_2}\mathbf{a}_{\rm rel}.$$

The forces toward the first and toward the second body are opposite

$$\mathbf{f}_1 = m_1\mathbf{a}_1 = -\frac{m_1 m_2}{m_1+m_2}\mathbf{a}_{\rm rel}, \quad \mathbf{f}_2 = m_2\mathbf{a}_2 = \frac{m_1 m_2}{m_1+m_2}\mathbf{a}_{\rm rel}.$$

If the space displacement is not admitted. Then it appears a relative rotation of the two bodies which is given by

$$\mathbf{w}_{\rm rel} = -\frac{v_0}{2} \sin\frac{r}{R_1} \sin\frac{r}{R_2} \frac{R_1 R_2}{r^2} \frac{\mathbf{r}}{r^2}. \tag{7}$$

Moreover the two bodies obtain opposite angular momentums

$$\mathbf{L}_1 = I_1\mathbf{w}_1 = -\frac{I_1 I_2}{I_1+I_2}\mathbf{w}_{\rm rel}, \quad \mathbf{L}_2 = I_2\mathbf{w}_2 = \frac{I_1 I_2}{I_1+I_2}\mathbf{w}_{\rm rel},$$

where I_1 and I_2 are the moments of inertia of the two bodies.

The previous formulas can be applied more generally, for example the first body can be a galaxy with radius of range R_1. If the second body is any star from the galaxy, then we put $R_2 = \infty$ and the previous formulas can be applied. As a consequence it is obtained ([15]) that it is not necessary to introduce dark matter, because the unknown force is just the strong force toward the center of the galaxy. The radius of range for the Milky Way is 17 kpc, which is twice longer than the distance from the Sun to the center of the galaxy. In case of the nucleons, the radius of range is about 1.41 fm.

3.2 Electromagnetic and electro-weak interaction

Let us consider two charged bodies with charges e_1 and e_2 and centers at O_1 and O_2, and let the second body has mass m_2. In the papers [14, 15] it is axiomatically assumed that the first body rotates the second body for angle

$$\vec{\theta} = \frac{e_1 e_2}{4\pi\epsilon_0 r^2 m_2 c^2}\mathbf{r}, \tag{8}$$

around the radial direction $\mathbf{r} = (x, y, z) = \overrightarrow{O_1 O_2}$. The angle θ has a physical interpretation as a potential, similar to the gravitational potential. Let τ be a translation for a small vector (a, b, c). We choose the coordinate system such that the angle of rotation is $(0, 0, \theta)$, i.e $x = y = 0$. Then the non-commutativity between the rotation for angle $\vec{\theta}$ and translation for vector (a, b, c) in the group $\mathcal{A}$ leads to translation of the point O_2 for vector $\overrightarrow{O_2O_2}' = (a(\cos\theta - 1) - b\sin\theta, a\sin\theta + b(\cos\theta - 1), 0)$. Indeed, it obtains by the following procedure: First rotation for angle θ, then translation τ, then rotation for angle $-\theta$ and then translation τ. This translation leads to the angle

$$\overrightarrow{O_2O_1O_2}' = (a(\cos\theta - 1) - b\sin\theta, a\sin\theta + b(\cos\theta - 1), 0)/r.$$

The unadmitted translation leads to the Coulomb's acceleration/force

$$\mathbf{a} = \frac{c^2}{2}\text{rot}\,\left(\frac{1}{r}(a(\cos\theta - 1) - b\sin\theta, a\sin\theta + b(\cos\theta - 1), 0)\right) = (\sin\theta)c^2\frac{\mathbf{r}}{r^2},$$

$$\mathbf{a} = \frac{e_1e_2}{4\pi\epsilon_0 r^3 m_2}\mathbf{r}, \quad \mathbf{f} = \frac{e_1e_2}{4\pi\epsilon_0 r^3}\mathbf{r}, \tag{9}$$

and so the electric field caused by the first charged body is

$$\mathbf{E} = \frac{e_1}{4\pi\epsilon_0 r^3}\mathbf{r}. \tag{10}$$

The induced angular velocities also appear.

Further let obtain the electro-weak interaction. Let us consider two charged particles with centers at O_1 and O_2, radii of range R_1 and R_2 and the coefficients k_1 and k_1^* have the same meaning as previously, and let (a, b, c) be a small vector of translation. Only the charged particles cause rotation. The rotations remain unchanged in both cases as in case of electromagnetic interaction, but there is change in the vector (a, b, c). Without loss of generality assume that the vector $\mathbf{r}$ is parallel to the z-axis, i.e. $x = y = 0$. Then the vector (a, b, c) from the first particle is observed as (k_1a, k_1b, k_1^*c). Although the basic group is G_s, the calculations are analogous as in $\mathcal{A}$. In this case we have translation for vector

$$\overrightarrow{O_2O_2}' = (k_1a(\cos\theta - 1) - k_1b\sin\theta, k_1a\sin\theta + k_1b(\cos\theta - 1), 0)$$

and it corresponds to angle

$$\angle\overrightarrow{O_2O_1O_2}' = (k_1a(\cos\theta - 1) - k_1b\sin\theta, k_1a\sin\theta k_1b(\cos\theta - 1), 0)/r.$$

The unadmitted translation leads to the acceleration/force of the second body toward the first body

$$\mathbf{a} = \frac{v_0^2}{2}\text{rot}\,\left(\frac{1}{r}(k_1a(\cos\theta - 1) - k_1b\sin\theta, k_1a\sin\theta + k_1b(\cos\theta - 1), 0)\right),$$

$$\mathbf{a} = \frac{v_0^2}{c^2}\left(\frac{R_1}{r}\sin\frac{r}{R_1}\right)\frac{e_1e_2}{4\pi\epsilon_0 r^3 m_2}\mathbf{r}, \tag{11}$$

$$\mathbf{f} = \frac{v_0^2}{c^2}\left(\frac{R_1}{r}\sin\frac{r}{R_1}\right)\frac{e_1e_2}{4\pi\epsilon_0 r^3}\mathbf{r}. \tag{12}$$

Symmetrically, the force of the first charged body toward the second charged body is given by

$$\mathbf{f} = -\frac{v_0^2}{c^2}\Big(\frac{R_2}{r}\sin\frac{r}{R_2}\Big)\frac{e_1e_2}{4\pi\epsilon_0 r^3}\mathbf{r}. \tag{13}$$

If $R_1 \neq R_2$, then these two forces are not opposite, and the symmetry is broken now. In a special case, when the mutual distance r between the two charged bodies is very close to 0 and v_0 is close to c, i.e. in case of high energies, then the weak interaction leads to the electromagnetic interaction.

Analogously to the strong interaction, the following angular velocity appears now

$$\mathbf{w} = \frac{v_0}{c^2}\Big(\frac{R_1}{r}\sin\frac{r}{R_1}\Big)\frac{e_1e_2}{4\pi\epsilon_0 r^3 m_2}\mathbf{r}. \tag{14}$$

The electro-weak and electromagnetic interaction can not occur simultaneously, but when the distance between the particles increases, the electro-weak interaction transforms into electromagnetic interaction.

3.3 Gravitational and gravity-weak interaction.

While the charged bodies mutually rotate for an angle θ determined by the axis of their centers, in case of gravitation we have radial translation from the center of the gravitational body. In case of gravitation, the translation refers to all points, while in case of charged bodies both bodies must be charged, i.e. the un-charged bodies are not rotated. This axiomatic gravitational translation has magnitude MG/c^2, where M is the mass of the gravitational body.

This gravitational translation combines with translation for a small vector $\tau = (a, b, c)$, and the non-commutativity leads to the gravitational acceleration. Assume that a point X has a radius vector $\mathbf{r} = (x, y, z)$, while the point-mass is at $(0, 0, 0)$. If we apply first translation for vector τ and then gravitational translation, we obtain

$$X(x,y,z) \to (x+a, y+b, z+c) \to Y\Big((x+a, y+b, z+c)\Big(1+\frac{GM}{r'c^2}\Big)\Big),$$

where $r' = r$ if we neglect the small lengths a, b, c. If we apply gravitational translation and then translation for vector τ, we obtain

$$X(x,y,z) \to (x,y,z)(1+\frac{GM}{rc^2}) \to Y'\Big((x,y,z)\Big(1+\frac{GM}{rc^2}\Big)+(a,b,c)\Big).$$

The non-commutativity of both translations gives an oriented angle

$$\angle\overrightarrow{YOY'} = \frac{\overrightarrow{OY}\times\overrightarrow{OY'}}{|OY|\cdot|OY'|} = -\frac{GM}{rc^2}\frac{(yc-zb, za-xc, xb-ya)}{r(r+GM/c^2)}.$$

Half of this angle is not admitted and it induces acceleration given by

$$\mathbf{g} = \frac{c^2}{2}\mathrm{rot}\angle\overrightarrow{BOB'} = -\mathbf{r}\frac{GM}{r^2(r+GM/c^2)} \approx -\mathbf{r}\frac{GM}{r^3}. \tag{15}$$

The other half which is admitted induces the known precessions in gravitational field for moving test body.

Analogously to the electro-weak, we have also gravity-weak interaction. It is close to gravitation via the group $\mathcal{A}$, and the observer will be a particle with radius of range R_1. Without loss of generality assume that the z-axis is parallel to the vector $\mathbf{r} = (x, y, z)$. Then the coordinates x, y, z, a and b should be multiplied by $k_1 = \frac{R_1}{r}\sin\frac{r}{R_1}$, while c should be multiplied by $k_1^* = \cos\frac{r}{R_1}$. It should be replaced into the coordinates of Y and Y', while the vectors $\overrightarrow{OY}$ and $\overrightarrow{OY'}$ should be divided by $(r + \frac{GM}{c^2})$. The calculation shows that the required acceleration is

$$\mathbf{g} = \frac{v_0^2}{2}\mathrm{rot}\angle\overrightarrow{YOY'} = -\frac{v_0^2}{c^2}\Big(\frac{R_1}{r}\sin\frac{r}{R_1}\Big)^2\frac{GM}{r^2(r+GM/c^2)}\mathbf{r}. \qquad (16)$$

Analogously to the strong interaction, the we have the following angular velocity

$$\mathbf{w} = -\frac{v_0}{c^2}\Big(\frac{R_1}{r}\sin\frac{r}{R_1}\Big)^2\frac{GM}{r^2(r+GM/c^2)}\mathbf{r} \approx -\frac{v_0}{c^2}\Big(\frac{R_1}{r}\sin\frac{r}{R_1}\Big)^2\frac{GM}{r^3}\mathbf{r}. \qquad (17)$$

In case of gravity-weak interaction the symmetry is also broken. The gravity-weak interaction is much smaller than the gravitational interaction and it is unknown.

The gravity-weak and gravitational interaction can not occur simultaneously, but when the distance between the particles increases, the gravity-weak interaction transforms into gravitational interaction.

4 Some results and comments

It is interesting that R.Boscovich in his ref. [1] considered also 4 basic cases between the space and time which are related to one point and analogous to them also 4 cases which are related for several points. He also comments which combinations, i.e. compositions among these cases are possible and which are not possible. There is an interesting analogy between his comments and the previous results.

The strong, weak and electromagnetic interactions are studied in the Standard Model. It is based on the Klein-Gordon equation and Dirac equation, from the Relativistic quantum theory [17]. While the non-relativistic theory starts from the formula $E = p^2/(2m)$, the Klein-Gordon equations starts from the corresponding relativistic formula $E^2 = p^2c^2 + m^2c^4$. The best experimental results in this direction are obtained by the Quantum Electrodynamics. Probably the reason is that the electromagnetic interactions belongs to temporal interactions, and the relativistic approach is convenient. According to this viewpoint the strong and weak interactions are not convenient to be researched by the same equation. For example, we can start from ([15]) $\mathbf{a} = \pm v_0\mathbf{w}$ (Theorem 1). Hence it follows that $a^2 = v_0^2w^2$ and by multiplication with m^2d^2 where m is the mass of the particle and d is a distance, we obtain $E^2 = L^2w^2$ where

$L = mdv_0$ is the angular momentum. The equation is $(E - Lw)(E + Lw) = 0$, and the sign $\pm$ in eq. $E = \pm Lw$ depends of the sign of the spin of the particle. It is analogous to the energy of the photon $E = \hbar w = h\nu$. Using wave theory the standard physical theory gives much attention of the interaction between the photon and the matter, and there Quantum Electrodynamics gives the best results. The results in section 3 are complementary, because in this paper we start from geometry and ignore the quantum and wave theory. The most important assumption is that we use the elementary particles as solid bodies, and the introduced small vector (a, b, c) can not be constrained. So we may conclude that if we consider the particles from wave theory, known theory should be applied, while if we consider as solid body, this model gives description of the interactions. However, the particles have dual nature.

To the end of this paper we consider some consequences from the previous results. Indeed, we start from the Lorentz invariance of the interactions. First we given some preliminaries which come from the metric in the 3-dimensional time. In the 3-dimensional time we distinguish two cases: i) the metric in the 6-dimensional space-time in $\mathbf{r}_s \times \mathbf{r}_t$, where the motion with velocity interpretes simply by rotating for an imaginary angle [3, 2], and ii) in case of active motion. In case i) the distance from a moving inertial coordinate system observes the length

$$|\Delta \mathbf{r}_s| = |\Delta \mathbf{r}| \sqrt{1 + \frac{\frac{v^2}{c^2} \sin^2 \psi}{1 - \frac{v^2}{c^2}}}, \tag{18}$$

where ψ is the angle between $\mathbf{v}$ and $\Delta \mathbf{r}$. In case of active motion, all lengths, i.e. in each directions should be multiplied by $\sqrt{1 - v^2/c^2}$. The last step (active motion) is a consequence since the time in moving systems changes, which is a subjective observation. Note that in case of both cases i) and ii) we obtain the same observation known from the Special Relativity. But since the active motion includes a subjectivity, we will use only the case i). According to this observation, each length in the direction of motion ($\psi = 0$) remains unchanged, while each direction which is orthogonal to the direction of motion ($\psi = \pi/2$), observes that the length is enlarged for coefficient $1/\sqrt{1 - \frac{v^2}{c^2}}$.

Now let us return to the Lorentz invariance. In case of the strong and weak interactions, the Lorentz covariance need not to be considered because the interactions are inside the group G_s. Let us start with the gravitational interaction. Assume that the observer is moving with velocity v with respect to the system in which the mass is in rest. In this case it is convenient to choose motion in a direction, such that the mass and the point which is translated for vector GM/c^2 are simultaneous. It occurs when the velocity is orthogonal to these two points. The Lorentz invariance means that the coefficients $\frac{GM}{c^2} : r$ must be preserved for the local observer and for the moving observer. It means that

$$\frac{GM}{c^2} : r = \frac{GM'}{c^2} : r'$$

where the mass M and the distance r are observed as M' and r' according to the moving observer. Since $r' = r/\sqrt{1-\frac{v^2}{c^2}}$, we obtain that $M' = M/\sqrt{1-\frac{v^2}{c^2}}$. This is known result, but accepted ad hoc without proof. It is suggested since the kinetic energy $mv^2/2$ can be written more precisely as $m(1/\sqrt{1-\frac{v^2}{c^2}}-1)$. Now, when we have a precise definition of the mass, we have also a precise proof.

Future let us consider two charged bodies with charges e_1 and e_2 which mutually rest on a distance r, and let the mass in rest of the second body be m_2. Then the second body is rotated for an angle (8). Let us choose an observer who moves with velocity v. Since the rotation is in the plane which is orthogonal to the axis which connects the two bodies, the observer should move in this direction. Otherwise he observes the angle as a part of ellipse. Let he observes the charges as e_1' and e_2'. Since the angle θ must be the same for both observers, we obtain

$$\frac{e_1 e_2}{4\pi\epsilon_0 r m_2 c^2} = \frac{e_1' e_2'}{4\pi\epsilon_0 r' m_2 c^2}.$$

Since $r' = r$ according to (18), we obtain $e_1 e_2 = e_1' e_2'$. If $e_1' = ke_1$, and hence also $e_2' = ke_2$, we obtain $k^2 = 1$, and hence $k = 1$ because $k > 0$. Thus we come to the known conclusion that the charge is observed unchanged in all inertial coordinate systems. It is accepted ad hoc without proof, according to the experiments.

Theorem 2. *The mass is observed enlarged for coefficient* $1/\sqrt{1-\frac{v^2}{c^2}}$ *while the charges is observed unchanged according to the observer who moves with velocity* $\mathbf{v}$.

References

[1] Bošković R. O prostoru vremenu i relativnosti. Kultura, Beograd; 1956. (original: De spatio, ac tempore. Ut a nobis cognoscuntur. De motu absoluto, an possit a relativo distingui. De vi inertiae.)

[2] K. Trenčevski, Duality in the special relativity based on the isomorphic structural groups $SO(3,\mathbb{C})$ and $O_+^\uparrow(1,3)$, Tensor 72(1) (2010) 32–46.

[3] K. Trenčevski, Special Relativity Based on the $SO(3,C)$ Structural Group and 3-dimensional Time, Mathematica Balkanica 25(1-2) (2011) 193–201.

[4] K. Trenčevski, Representation of the Lorentz transformations in 6-dimensional space-time, Kragujevac J. Math. 35(2) (2011) 327–340.

[5] K. Trenčevski, On the group of isometries of the space, (BSG Conf. Proc. 21) (eds) C. Udriste and V. Balan (Romania: BSG) (2014) p. 193–200.

[6] A. Yefremov, Bi-quaternion square roots, rotational relativity, and dual space-time intervals, Gravitation and Cosmology 13(3) (2007) 178–184.

[7] A. Yefremov, Notes on pioneer anomaly explanation by satellite shift formula of Quantum Relativity: Remarks on less mundane explanation on Pioneer anomaly from Q-Relativity, Progress in Physics 2 (2007) 93.

[8] V. S. Barashenkov, Quantum field theory with three-dimensional vector time, Particles and Nuclei, Letters 2 (2004) 54–63.

[9] X. Cheng, Three dimensional time theory: to unify the principles of basic quantum physics and relativity, (2005) arXiv: quant-ph/0510010.

[10] X. Cheng, Modified Kaluza-Klein theory, quantum hidden variables and 3-dimensional time (2005) arXiv: quant-ph/0501034.

[11] H. Kitada, L. R. Fletcher, Unification of physics part I. Local time. Apeiron 3(2) (1996) 38–45. (arxiv: gr-qc/0110065)

[12] H. Kitada, Three dimensional time and energy operators and uncertainty relation, (2000) arXiv: quant-ph/0007028.

[13] M. Tegmark, On the dimensionality of the spacetime, Classical and Quantum Gravity 14 (1997) L69–L75.

[14] K. Trenčevski, Geometrical aspects of the electromagntic field, Chapter in "Electric field in advancing sciences and technology", IntechOpen, London, ISBN 978-1-80356-678-8 (in press), (intechopen.com/online-first/83987).

[15] K. Trenčvski, Global scheme of the basic interactions and their geometrical interpretations, Filomat, accepted for publication.

[16] K. Trenčevski, E. Celakoska, Induced spin velocity of the Earth and its influence to the seasonal variation of the Earth's angular velocity, Eur. Phys. J. Plus (2020) 135:450.

[17] N.Zovko, Osnove Relativističke kvantne fizike, Školska Knjiga, Zagreb 1987.