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PROMOTING REFLECTION DURING MENTAL MATH

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Abstract. Textual tasks, magic squares, mathematical riddles, puzzles and rebuses can be used as a tool for motivating and encouraging students' interest in mathematics. The aim of this paper is to suggest several such tasks that increase students' interest in mathematics and mathematical challenges, deepen their knowledge and encourage the process of acquiring long-lasting and structural knowledge among students.
Simultaneously, by pointing and discussing solutions we can increase students' mathematical thinking, promoting math reflection and encourage them to think about formulating new tasks that will have similar or different solutions.

1. INTRODUCTION

Physical activity is essential for the healthy and proper development of our body, in that direction the statement "healthy body healthy mind" is generally accepted. However, what do we know about the food for our mind? What do we need to do to train the mind and encourage logical thinking? What is mathematical thinking? How do we help the process of its formation?

Mathematical tasks and solving them are the "healthy food" for the mind. In this presentation, we will pay special attention to mathematical puzzles, logical tasks and games, textual tasks, tasks with strategies, symmetries, and tasks with tricks, i.e., different tasks that develop thinking and maintain our mind in perfect condition. It is important to note that the process of development and formation of mathematical concepts, development of thinking abilities, rational approach to problem solving, implementation of different ways of solving problems are development processes that can be assisted and guided. With appropriate, interesting tasks, we will try to "stimulate the process of thinking" and to manage the procedures necessary for solving the assigned tasks. In order to achieve maximum knowledge of every gifted and non-gifted student it is necessary to intensively form some kind of an ability during the sensitive period (period of highest receptiveness and development). The thinking process in a person begins when an intellectual or practical task emerges. Therefore, the qualities of thinking develop precisely while solving such tasks, and in school, they are posed to students not in a chaotic manner, but in a determined system of teaching tasks.

Teaching tasks develop students' thinking when well motivated, well understood, appropriate to the achieved level of intellectually, related to life problems. As stated above, thinking is a process of complex information processing, the end-result is "concepts-words" and "thinking-sentences." The teaching process involves empirical and theoretical thinking because it is a complete

cognitive process through which students acquire the social and historical experience of humanity. Understanding that students adopt this experience in a short, generalised, structured and systemised form during the process of learning, is a problem for the corresponding part of empirical and theoretical thinking of learning through which this basic task of schools can be achieved.

All of this is in the function of improving the understanding and application of mathematical knowledge and skills, creating lasting knowledge and encouraging curiosity and love of mathematics. Given examples of tasks will encourage students not only to think, but also to want new mathematical challenges and new tasks.

The thesis of the interaction of the different types of thinking in the process of solving the teaching tasks does not contradict, but supports the character of the school activity as a complex scientific activity. Solving each teaching task requires not only simple processing of sensory material, confirming the facts (empirical opinion), but also getting to the essence of the situation of the relationships between them through specific complex mental activities for theoretical thinking, such as: creating situations in order to discover the common relationships in the studied system; modelling these relationships into graphic signs and forms in order to be studied in the same form, building a series of tasks of a general manner of solving; controlling and evaluating the manners to solve the teaching task.

Thinking is a process of complex information processing, the end-result is "concepts-words" and "thinking-sentences." Understanding is of great importance when trying to solve a teaching task. It is an active, multifaceted cognitive activity aimed at stating and discovering abstract connections between the new in the task and the imminent from the various subsystems of knowledge. As a result of this process, a new configuration of subsystems is born, which recognises (aha-moment), the adoption of the new from the given task.

Development of thinking and in particular the development of mental qualities — width, depth, independence, logic, mobility, concreteness, criticism, speed, creativity, target orientation, generalisation, insight, etc., . , is one of the most important and consistent goals and objectives of the teaching.

School mathematics, due to its specificity, possesses great opportunities for scholar’s intellectual development which can be fully accomplished through prior organisation of the educational process. From this point of view, the conclusion of Vygotsky - Leontev’s school of psychology according to which the child’s development occurs in a process of adopting historically created mathematical knowledge, skills and habits is extremely important.

Mathematical tasks, the moment they enter the classroom, are intertwined with the educational aims, intentions and interactions between the teachers and the students. Therefore, tasks should not be considered as problems written in math text books or in the teachers’ preparation, but should be considered as a classroom activity as well. Defined as activities, mathematical tasks in the

educational process become connected and included in both training and teaching.

2. WHY DO WE NEED MENTAL MATH AND MENTAL MATH REFLECTION?

During mental math students

- Encourage mathematical thinking
- Stimulate curious children who want to explore
- Provide a mathematical record of problem situations and formation of mathematical models
- Foster love for mathematics and problem situations
- Encourage interest in challenges and assessments
- Stimulate creative thinking and students' motivation in they're learning
- Acquire long-lasting, structural knowledge among students

1. Continue the number sequence and determine the seventh term:

a) 1,2,3,4,….

b) 11,12,13,14, …

c) 1,1,2,2,3,3,4,4,5,5,

d) 2,4,6,8,10…

e) 1,3,5,7,9…

2. Continue the number sequences with one of the offered possible answers and explain your choice:

a) 1, 4, 10, 22, 46,…

Possible answers: 64, 86, 94, 122

b) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$

Possible answers: $\frac{1}{14}, \frac{1}{4}, \frac{1}{12}, \frac{1}{8}$

3. Solve the rebus if same letters denote same digits:

$$\begin{array}{r} ЦАР \\ +\ ЦАР \\ \hline КРАЛ \end{array}$$

How many solutions are possible? Explain!

4. Solve the rebus if same letters denote same digits:

$$\begin{array}{r} A \\ BB \\ +\ \ A \\ \hline CCC \end{array}$$

5. There are two glasses with the same amount of white and red wine. We pour one-tenth of the red into the white wine, and then one-tenth of the resulting mixture back into the red wine.

Is there more white wine into the red wine or vice versa? Explain!

6. Three containers with no markings have volumes of 8, 5 and 3 l and the largest container is full. How do you measure an amount of 4 litres using the given containers?

3. EXAMPLES OF TASKS AS AN INTRODUCTION TO TASKS REDUCED TO IMPLEMENTATION OF LINEAR EQUATION

1. Which number is 4 times greater than 16?
2. 5 balls cost 85 denars. How much does one ball cost?
3. Nela has 564 denars. She has 236 denars more than her mother. She wants to buy a bag that costs 1000 denars. How much more money does she need?
4. 246 letters and 85 more postcards than letters were received in the post office. 110 fewer packages than letters were received. How many total items (letters, postcards and packages) were received in the post office?
5. Calculate the sum of the number 5 and its predecessor.
6. Find the difference between the number 9 and the predecessor of 3.
7. The sum of two numbers is 94. The larger number is 5 less than twice the smaller number. Find the two numbers.
8. Jana is 7 years older than her brother. In 5 years, the sum f their ages is 63. How old is each of them?
9. Petar had 6500 denars in banknotes of 50, 100 and 500 denars. He had an equal number of banknotes of every kind. How many banknotes of each kind did he has?
10. In a supermarket, the prices of food products decreased by 40%. If the price of a product is 180 denars, determine its price before the discount.
11. To build a fence around a rectangular garden requires 130 meters of wire. The length of the garden is 5 m larger than the width. Determine the dimensions of the garden.
12. A cyclist traveled 18 km during which time a pedestrian traveled 10 km If the cyclist was traveling at a speed of 9.6 km/h greater than the speed at which the pedestrian traveled, at what speed did each of them travel?
13. Two farmers can plow a field in 6 days. The first farmer can plow the field alone in 10 days. How many days will the second farmer need to plow the field alone?
14. The number 48 should be divided into two parts so that one part is three times larger than the other.
15. The numerator of a fraction is 2 less than the denominator. If the numerator decreases by 1 and the denominator increases by 1, the fraction ½ is obtained. Find the initial fraction.
16. The sum of three numbers is 54. Find the numbers if the first number is 4 larger than the doubled value of the second number and the third number is twice the first number.
17. In a mathematics test, the student must solve 20 tasks. The student receives 4 points for each task that is correct, and for each unsolved or

incorrectly solved task he loses 3 points. On that test, the student won a total of 38 points. How many tasks did the student solve correctly?

18. A worker can complete a job in 12 days. After working on that job for 3 days, another worker started to help who could complete the whole job in 15 days. How many days will they need to complete the job?

19. A man went on a journey by walking 30 km a day. 6 days later, another man followed the same route and after 9 days reached the first traveler. At what speed was the second traveler moving?

20. Sally is having a party. The first time the doorbell rings, 1 guest enters. The second time the doorbell rings, 3 guests enter. The third time the doorbell rings, 5 guests enter. Keep going in the same way. On the next ring a group enters that has 2 more persons than the group that entered on the previous ring.

A. How many guests will enter on the 5 th ring? Explain or show how you found your answer
B. How many guests will enter on the 10th ring? Explain or show how you found your answer.
C. 19 guests entered on one of the rings. What ring was it?

New goals for students who solve logical tasks

(1) They learn to value mathematics
(2) They become confident in their ability to do mathematics
(3) They become mathematical problem solvers
(4) They learn to communicate mathematically
(5) They learn to reason mathematically

4. STAGES IN SOLVING LOGICAL TASKS

Stage 1: Understand the task (read the entire task or parts of it, drawing, sketch, symbolic representation of the task). This is the invisible stage and teachers usually skip it.

Stage 2: Build an idea and devise a plan to solve the task (this stage is connected with understanding the task).

Stage 3: Practical implementation of the devised plan (mathematical operations and solving the equation).

Stage 4: Examine the obtained solution (creative and interesting questions related to the task are asked additionally) such as: Is the obtained result correct? Why?

It is desirable to include as many students as possible in the classes, first using individual method of solving tasks and then making discussion. Namely, the previously stated and similar examples provide students with the opportunity to obtain not only wide-ranging operational knowledge and skills, but creating long-lasting structural knowledge. Increased level of this type of knowledge is, above all, conditioned by the fast-growing need for comprehensive and profound knowledge necessary to keep the active pace with the dynamic civilisation of the twenty first century.

CONCLUSIONS

Mathematical tasks and solving the same are an effective tool for developing mathematical activity and creativity among students. Reflection is associated with the "aha" moment, and it is necessary for creating long-lasting, structural knowledge among students. In this paper, appropriate tasks were selected which will further stimulate and motivate math reflection. I hope that this paper in which textual tasks, magic squares, mathematical riddles, puzzles and rebuses are used as a tool for motivating and encouraging students' interest in mathematics will motivate many teachers and contribute to maintaining mathematical talent among students and increasing love and interest in mathematics.

References

[1] Georgieva M.: Reflection in Mathematics Education (V-VI grade), Veliko Trnovo 2001

[2] Gogovska, V., Anevska, K., Malceski, R. *Properties of Thinking and Adoption of Mathematical Knowledge* , Proceedings of the 2014 International Conference on Education and Educational Technologies II (EET '14) , 101-105

[3] Grozdev,S. (2007). For High Achievements in Mathematics. The Bulgarian Experience (Theory and Practice), Sofia:

[4] Jankovic, M., Kovacevik, S. Matematicarenje, za ucenike iv razreda osnovne skole, Pcelica, Cacak 2017

[5] Malcheski, R. (2012) “Didactic of mathematics teaching (basic part)”, FON University, Skopje.

[6] Sfard, A. (2005). What could be more practical than a good research? On mutual relations between research and practice of mathematics education (Survey Team, 1 Report on Relations between research and Practice of Mathematics Education). Educational Studies in Mathematics 58, 393–413

[7] Stain.M.K., Smith M.S., Henningsen M.A., Silver E.A., Implementing Standard – based on Mathematics Instructions, NCTM, USA, 2007

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