Union of Mathematicians of Macedonia - ARMAGANKA

IX SEMINAR OF DIFFERENTIAL EQUATIONS AND ANALYSIS

and

1st CONGRESS OF DIFFERENTIAL EQUATIONS, MATHEMATICAL ANALYSIS AND APPLICATIONS

CODEMA 2020

Proceedings of the CODEMA 2020
Зборник на трудови од CODEMA 2020

Skopje, 2021

REGRESSION METHODS IN ANALYSIS OF GEOTECHNICAL PARAMETERS OF COAL DEPOSITS

ISBN 978-608-4904-09-0 UDC: 519.242:553.94.048

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Abstract. In this paper, a comparative analysis of geotechnical parameters of coal deposits from different sedimentation basins in Macedonia is given. The knowledge of the geotechnical properties of the rock masses in mineral deposits is very important for the economy of the extraction process, safety of work and protection of the environment. From the geological and geotechnical investigations of the coal deposits, usually a relatively large stock of data for the physical and mechanical properties of materials is obtained, forming a series that can be statistically analyzed. Analysis of selected samples and types of performed tests for several deposits was conducted, as well as a comparative analysis of the values for the following geotechnical parameters: natural moisture on each deposit, unit weight and coefficients of filtration. Several correlation dependencies have been established between certain geotechnical parameters, such as: unit weight - dry unit weight and porosity - filtration coefficient, for which high values for the correlation coefficient R^2 (which indicates a strong to very strong dependance) are obtained. With such analyses, a procedure for processing a larger pool of data from geotechnical investigations of mineral deposits is defined. The introduction of such a procedure aims in selecting and adopting the most relevant value for certain geotechnical parameter, and generally define reliable geotechnical parameters for coal deposits, which in turn are needed to perform a number of other geotechnical (both geological and mining) analyses.

1. INTRODUCTION

This paper is devoted on coal deposits and establishing a procedure for defining their most realistic geotechnical parameters. The importance of complete information for the geotechnical parameters is seen both in economic terms and safety of the extraction process, as well as environmental protection. Whether the coals are excavated underground or on the surface, it is always necessary to have detailed knowledge on their geotechnical parameters in order to have good implementation of the processes of planning, designing, exploitation and closing of coal mines. The coal is one of the main source of energy in our country, and in the last four decades some serious activities in coal's investigations, including geotechnical activities, are conducted.

2010 *Mathematics Subject Classification.* Primary: 62P30, Secondary: 65C20, 65C60.
Key words and phrases.coal deposit, geotechnical parameters, statistical analysis, regression, comparative, correlations.

The data of the state's electricity production company show that there are total confirmed reserves of 664 million tons of coal. This fact implies that there is strong need of formation of clear and detailed geotechnical profiles of the deposits, with emphasis on possible excavation depths in underground excavation or high slopes in the case of surface excavation. Therefore, the need of knowledge about the differences in geotechnical parameters in mineral deposits from different sedimentation basins and specific correlations used in different phases of design and exploitation of the coal, is quite strong.

Herein is presented analyzed data from four coal deposits. More precisely, data on natural moisture content, unit weights, and coefficient of filtration is considered. Then a correlation analysis between unit weight-dry unit weight and porosity-coefficient of filtration for certain materials within the coal deposits is conducted. As main and ultimate goal of this paper is to use regression method for proper selection and adoption of most representative values of specific geotechnical parameters, which can be used for further geotechnical analysis. Another goal in this paper is to perceive the influencing factors on the parameters, to compare and make possible correlations.

From all coal reserves, it is estimated that around 38% can be exploited with surface excavation, and the rest with underground technology (underground coal excavation in our country is still not applied). Having on mind that the surface excavation is the usual way of excavation in our country, it is necessary to have surface mines with relatively great depths. This includes design of stabile slopes, which on the other hand implies need for detailed picture for the geological and geotechnical profile of the coal deposit. Hence, a complex geological, engineering-geological, hydro-geological and geotechnical investigations, consisting of drilling exploratory boreholes, field and laboratory tests, have to be done.

For the purpose of the comparative analysis of the geotechnical parameters of coal deposits of different sedimentation basins and available data, a choice of certain mineral deposits, who will be considered in this paper is made. The chosen coal deposits are:

- Suvodol and Brod-Gneotino (part of the Pelagonia basin)
- Lavci (part of Prespa basin)
- Zvegor-Stamer (part of Delchevo-Pehchevo-Berovo's basin).

For all mentioned coal deposits, in different time periods, certain amount of geological and geotechnical, as well as laboratory works are made.

The data in this paper is taken from technical documentation prepared in the time period 1985 to 2018, and the number of the results for each coal deposit varies from 600 to 1200. We investigate only three parameters of the deposit materials in this paper. The large amount of data allows us to make statistical analysis with quite big confidentiality and also statistical comparison between different coal deposits. The comparative analysis is given by tables and graphs, usual tool of descriptive statistics, relieving the process of making conclusions. This type of approach allows to determinate the confidence interval for particular geotechnical parameter, to detect non logical values, to find maximal, average and minimal values, standard deviations, median and other statistical parameters for the materials in the mineral deposits. Using these type of analysis, we can deduce certain conclusions for geotechnical parameters of particular deposit/s, to plan future phases of investigations for each deposit and to prescribe the number of sufficient number of tests. Correlations between certain parameters and dependences with high coefficient of correlation are obtained.

These type of correlations enable efficient programmed investigations for further explorations and reducing of unnecessary costs in case of consistency of some specific parameter. Having all this on mind, it is quite clear that the results placed here could be used in coal exploration and exploitation practices.

2. METHODOLOGY

In this section are presented the main statistical definitions and methods used for analysis of the data obtained from coal deposits mentioned in the Introduction. It important to note that, to our knowledge, this kind of approach in analysis of geotechnical parameters of the coal deposits has never been done before.

The definitions and basic facts about average values, standard error, median, mode, standard deviation, variance are quite known statistical and very recently used notations in descriptive statistics, so we are going to omit them. We are going to present same basic facts about skewness, kurtosis, confidence intervals, boxplots, correlation and regression.

The skewness usually is described as a measure of the symmetry of data, or it can be interpreted as a lack of symmetry. Perfectly symmetric set of data have skewness 0. The formula calculating skewness is the following

$$a_3 = \sum \frac{(x_i - \bar{x})^3}{ns^3}.$$

Here we can notice that the exponent is 3, since the skewness is "the third standardized central moment in the probability".

The kurtosis is statistical measure defining how the tails (the parts which are far from the peaks, i.e. the left and the right ends of the curve) of the distribution differ from the tail of standard normal distribution. This means that the kurtosis gives us information whether the tails of certain distribution contains extreme values. The kurtosis does not study the peaks (extremes), curved and flat parts of the distribution. The kurtosis is given by the following

$$a_4 = \sum \frac{(x_i - \bar{x})^4}{ns^4}.$$

It can be noticed that the exponent is 4, since the kurtosis is "the fourth standardized central moment in the probability".

In the expressions for skewness and kurtosis, n is the size of data, x_i is the i-th value of the sample, $\bar{x}$ is the average value of the sample data and s is the standard deviation.

Usually confidence intervals are used when we want to give an estimate of some parameter of the population, i.e. how accurate is the estimate with certain probability. 95% confidence interval means that when we have repeatedly sampling (sample series) from the population, 95% of the obtained intervals will include the real value of the parameter of the investigated population. It is clear that bigger sample implies more reliable confidence interval.

The maximal and minimal observations tell us little about the distribution of the sample values, but they give us information about the tails of the distribution that are missing if we know only the median and the quartiles. To obtain a quick summary of both center and spread, we combine the following five numbers: minimum, first quartile, median, third quartile and maximum, written in order from smallest to largest. These five numbers offer a reasonably complete description of center and spread. A boxplot is a graph of the five-number summary: A central box spans the quartiles Q_1 and Q_3, a line in the box marks the median M, lines extend from the box out to the smallest and largest observations. Because boxplots show less detail than histograms or stemplots, they are best used for side-by-side comparison of more than one distribution.

The regression line summarizes the relationship between two variables, but only in a specific setting when one of the variables helps explain or predict the other. A regression line is a line that describes how a response variable y changes as an explanatory variable x changes. This line often is used to predict the value of y for a given value of x. The least squares regression line of y on x is the line that makes the sum of the squares of the vertical distances of the data points from the line as small as possible.

A quadratic regression is the process of finding the equation of the quadratic function (parabola) that best fits a set of samples. As a result, we get an equation of the form

$$y = ax^2 + bx + c, \qquad a \neq 0.$$

The best way to find this equation is by using the least squares method. That is, we need to find the values a, b and c such that squared vertical distances between each point (x_i, y_i) and the quadratic curve $y = ax^2 + bx + c$ is minimal. The equation of the curve of the regression curve can be found as a solution of the following system

$$\begin{cases} a \cdot \sum x_i^4 + b \cdot \sum x_i^3 + c \cdot \sum x_i^2 = \sum x_i^2 y_i \\ a \cdot \sum x_i^3 + b \cdot \sum x_i^2 + c \cdot \sum x_i = \sum x_i y_i \\ a \cdot \sum x_i^2 + b \cdot \sum x_i + n \cdot c = \sum y_i \end{cases}, \quad i = 1, 2, \ldots n.$$

The coefficient of correlation R^2, can be found by the following formula

$$R^2 = 1 - \frac{RSS}{TSS},$$

whereby RSS is denoted the sum of squares of residuals and by TSS is denoted the total sum of squares, i.e.

$$RSS = \sum\left(y_i - ax_i^2 - bx_i - c\right)^2 \text{ and } TSS = \sum\left(y_i - \bar{y}\right)^2 .$$

Correlation and regression are closely connected. It is important to note that we should be careful when we want to conclude that there is a cause-and-effect relationship between two variables just because they are strongly associated, i.e. high correlation does not imply causation.

According to the quantity of the coefficient of correlation (R^2) we can determine the strength of the regression, usually using the following criteria:

- if $R^2 < 0,3$ there isn't any dependence;
- if $0,3 \le R^2 < 0,5$ there is some dependence;
- if $0,5 \le R^2 < 0,7$ there is mild dependence;
- if $0,7 \le R^2 < 0,9$ there is strong dependence;
- if $R^2 \ge 0,9$ there is very strong dependence.

3. APPLICATIONS

In this part, comparative analysis for certain geotechnical parameters (natural moisture content, unit weight and coefficient of filtration) for certain type of geotechnical materials was made. There are given boxplot and certain tables for the geotechnical parameters for certain materials (average value, standard error, median, mode, standard deviation, variance, kurtosis, skewness, range, minimum, maximum, sum, sample numbers and confidence interval) and certain correlations between unit weight-dry weight of certain materials and correlations between porosity-coefficient of filtration for certain materials in coal deposits.

3.1. Natural moisture content

On the given boxplot above for each coal deposit are given minimal and maximal value of natural moisture, first and third quartile and median. For the coal deposits Suvodol, Lavci and Brod-Gneotino, minimal value of the moisture of the samples varies around 10% and maximal value varies around 50%. These values for the samples are quite different in the case of coal deposit Zvegor-Stamer, i.e. minimal value of the natural moisture around 5% and maximal value of the natural moisture content around 70%. Generally, we can note that in all investigated coal deposits, the samples have natural moisture content between 20 – 40% .

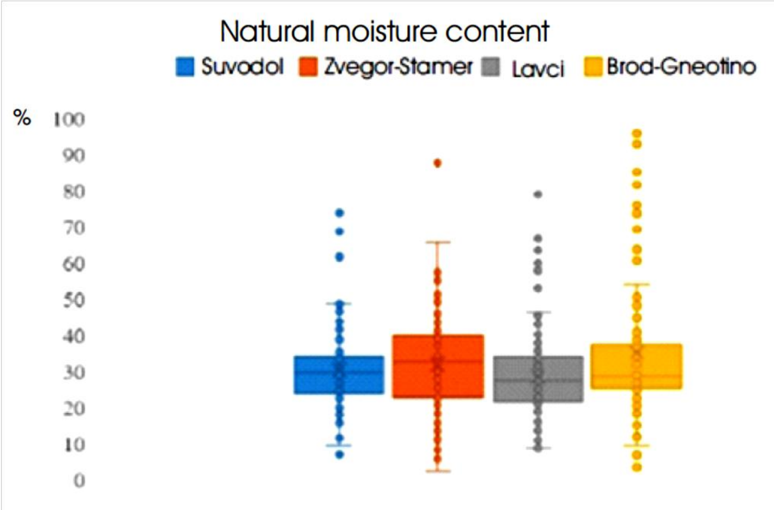

Figure 1. Natural moisture content of some deposits

3.2. Unit weight

Unit weight is one of the most often investigated geotechnical parameters in all deposits. This parameter is essential for every geotechnical analysis. The comparative analysis for all investigated coal deposits is given through the tables in sequel.

Table 1. Unit weight of gravels (in kN/m^3)

Gravels	Zvegor-Stamer	Lavci
Average value	19,20	20,48
Standard error	0,325	0,517
Median	19,205	20,2
Mode	/	/
Standard deviation	0,4596	1,2684
Variance	0,2112	1,6089
Kurtosis	/	–0,6372
Skewness	/	–0,0593
Range	0,65	3,37
Minimum	18,88	18,63
Maximum	19,53	22
Sum	38,41	122,89
Sample size	2	6
95% Confidence interval	4,1295	1,3311

From the table above, we can conclude that gravels are characterized with the close minimal value, but big difference in its maximal value $19{,}53\,\text{kN/m}^3$ in Zvegor-Stamer and $22{,}00\,\text{kN/m}^3$ in Lavci. Since this obvious difference, there is deviation in the average value, which is $19{,}21$ kN/m^3 and $20{,}48\,\text{kN/m}^3$, for Zvegor-Stamer and Lavci, respectively.

Table 2. Unit weight of sands (in kN/m^3)

Sands	**Zvegor-Stamer**	**Suvodol**	**Lavci**	**Brod-Gneotino**
Average value	18,98	19,48	19,73	18,55
Standard error	0,2992	0,1788	0,1374	0,2780
Median	19,18	19,565	19,985	18,72
Mode	/	19,81	19,8	/
Standard deviation	0,7916	1,2386	0,8687	1,3331
Variance	0,6267	1,5341	0,7547	1,7772
Kurtosis	0,4486	2,1985	3,5553	4,1310
Skewness	–1,0409	0,6187	–1,3222	–1,6424
Range	2,31	6,3	4,7	6,23
Minimum	17,57	16,9	16,5	14,23
Maximum	19,88	23,2	21,2	20,46
Sum	132,83	935,14	789,27	426,76
Sample size	7	48	40	23
95% Confidence interval	0,7321	0,3596	0,2778	0,5765

Minimal value of the unit weight of the sands from the coal deposits is the biggest from $17{,}57\,\text{kN/m}^3$ for Zvegor-Stamer and the least value of $14{,}23$ kN/m^3 for Brod-Gneotino. The maximal value varies in all coal deposits, i.e. it varies from $19{,}88\,\text{kN/m}^3$ to maximal value of $23{,}20\,\text{kN/m}^3$. The average values have small deviation, that is $18{,}55\,\text{kN/m}^3$ for Brod-Gneotino, $18{,}98\,\text{kN/m}^3$ for Zvegor-Stamer, $19{,}48\,\text{kN/m}^3$ for Suvodol and $19{,}73\,\text{kN/m}^3$ for Lavci. There is obvious difference in unit weight of sands at Suvodol and Brod-Genotino, although these coal deposits are in the same sedimentation basin.

Table 3. Unit weight of silt (in kN/m^3)

Silts	**Zvegor-Stamer**	**Suvodol**	**Lavci**	**Brod-Gneotino**
Average value	18,36	18,84	17,99	17,65
Standard error	0,29	0,0720	0,8851	0,3560
Median	18,36	18,88	17,905	18,24
Mode	/	18,94	/	17,86

Standard deviation	0,4101	0,4136	2,1682	2,4409
Variance	0,1682	0,1710	4,7009	5,9582
Kurtosis	/	3,0037	–0,8651	0,4249
Skewness	/	0,8640	0,3194	–0,8172
Range	0,58	2,13	5,91	11,97
Minimum	18,07	18,1	15,29	11,02
Maximum	18,65	20,23	21,2	22,99
Sum	36,72	621,92	107,94	829,74
Sample size	2	33	6	47
95% Confidence interval	3,6848	0,1466	2,2753	0,7167

Minimal values of the unit weight for silts are 18,07 kN/m^3 and 18,10 kN/m^3, in Zvegor-Stamer and Suvodol, respectively, 15,29 kN/m^3 in Lavci and the lowest value of 11,02 kN/m^3 in Brod-Gneotino. The maximal value is the lowest in Zvegor-Stamer with 18,65 kN/m^3 and the highest with 22,99 kN/m^3 in Brod-Gneotino. The average values are almost the same, i.e. for 17,99 kN/m^3 and 17,65 kN/m^3 for Lavci and Brod-Gneotino and higher, i.e. 18,36 kN/m^3 and 18,85 kN/m^3 for Zvegor-Stamer and Suvodol, respectively.

Table 4. Unit weight of clays (in kN/m^3)

Clays	**Zvegor-Stamer**	**Suvodol**	**Lavci**
Average value	17,81	18,37	18,73
Standard error	3,085	0,3563	0,2582
Median	17,815	18,6	19,065
Mode	/	/	18,74
Standard deviation	4,3628	0,8727	1,6330
Variance	19,0344	0,7615	2,6667
Kurtosis	/	–1,5238	2,7559
Skewness	/	–0,4795	–1,4517
Range	6,17	2,19	8,08
Minimum	14,73	17,21	13,02
Maximum	20,9	19,4	21,1
Sum	35,63	110,2	749,23
Sample size	2	6	40
95% Confidence interval	39,1986	0,9158	0,5223

It can be noticed differences in the values of minimal and maximal values of unit weight at clays in all deposits. In point of view of average values, there are relatively close values of 18,37 kN/m^3 and 18,73 kN/m^3 in Suvodol and Lavci, and lower value of 17,82 kN/m^3 for Zvegor-Stamer.

Table 5. Unit weight of organic clays (in kN/m^3)

Organic clays	**Suvodol**	**Lavci**	**Brod-Gneotino**
Average value	18,52	16,10	16,80
Standard error	0,1522	0,5828	0,6103
Median	18,73	15,78	17,01
Mode	20,07	/	/
Standard deviation	1,1493	1,3032	1,7263
Variance	1,3209	1,6983	2,9801
Kurtosis	2,2970	–1,1197	–1,4510
Skewness	0,7957	–0,3277	–0,3404
Range	6,8	3,13	4,73
Minimum	16,04	14,3	14,18
Maximum	22,84	17,43	18,91
Sum	1055,77	80,49	134,42
Sample size	57	5	8
95% Confidence interval	0,3050	1,6181	1,4432

For organic clays, the differences in values on minimum and maximum of unit weight at all deposits are obvious. There are also differences at average values, i.e. the lowest average value of 16,10 kN/m^3 for Lavci, then average value of 16,80 kN/m^3 for Brod-Gneotino and much higher average value in Suvodol of 18,52 kN/m^3. The difference in size of the analyzed data is obvious.

Table 6. Unit weight of alevrolites (in kN/m^3)

Aleurolites	**Zvegor-Stamer**	**Lavci**
Average value	16,56	19,52
Standard error	0,2180	0,1685
Median	16,62	19,525
Mode	16,37	/
Standard deviation	1,7843	0,6305
Variance	3,1837	0,3975
Kurtosis	0,1699	–0,9202
Skewness	–0,1516	0,2934
Range	8,2	2
Minimum	13,36	18,7
Maximum	20,56	20,7

Sum	1109,84	273,26
Sample size	67	14
95% Confidence interval	0,4352	0,3640

The alevrolites (type of sediment consisting of bonded grains of silt) are present only in Zvegor-Stamer and Lavci. They have very big difference in minimal values and almost same maximal values for unit weight. In average values, also is clear difference, that is 16,56 kN/m^3 in Zvegor-Stamer and 19,52 kN/m^3 in Lavci.

Table 7. Unit weight of coals (in kN/m^3)

Coals	**Zvegor-Stamer**	**Suvodol**	**Lavci**	**Brod-Gneotino**
Average value	14,21	12,41	13,97	11,88
Standard error	0,045	0,1318	1,4989	0,3992
Median	14,215	12,54	11,14	11,99
Mode	/	/	/	/
Standard deviation	0,0636	0,6040	3,9658	0,6916
Variance	0,0040	0,3648	15,7276	0,4783
Kurtosis	/	1,2736	–1,9778	/
Skewness	/	–0,9868	0,6376	–0,6976
Range	0,09	2,47	8,6	1,37
Minimum	14,17	10,75	10,61	11,14
Maximum	14,26	13,22	19,21	12,51
Sum	28,43	260,67	97,81	35,64
Sample size	2	21	7	3
95% Confidence interval	0,5718	0,2749	3,6678	1,7180

The differences of minimal, maximal and average values of unit weight for the coal of all analyzed deposits are obvious. According the average values, lowest unit weight of 11,88 kN/m^3 has the coal in Brod-Gneotino, then 12,41 kN/m^3 has the coal in Suvodol, 13,97 kN/m^3 has the coal in Lavci and the highest unit weight of 14,22 kN/m^3 has the coal in Zvegor-Stamer.

3.3. Coefficient of filtration

The coefficient of filtration is parameter which determines the water permeability of the local environment, i.e. the deposit's sediments and can be determined by field and laboratorial experiments, as well as by empirical equations. The subject of this analysis is laboratory obtained values.

Table 8. Coefficient of filtration (kf [cm/s)] of sands

Sands	**Zvegor-Stamer**	**Suvodol**	**Lavci**	**Brod-Gneotino**
Average value	$0,000126$	$0,00012$	$0,000842$	$0,002303$
Standard error	$5,44\cdot10^{-5}$	$5,07\cdot10^{-5}$	$0,000318$	$0,000492$
Median	$6,26\cdot10^{-5}$	$8,16\cdot10^{-5}$	$1,65\cdot10^{-5}$	$0,00078$
Mode	/	$8,16\cdot10^{-5}$	$0,003$	$0,00078$
Standard deviation	$0,000154$	$0,000209$	$0,001772$	$0,00023$
Variance	$2,37\cdot10^{-8}$	$4,36\cdot10^{-8}$	$3,14\cdot10^{-6}$	$0,002459$
Kurtosis	$-0,07193$	$13,47091$	$3,173536$	$6,05\cdot10^{-6}$
Skewness	$1,336927$	$3,526208$	$2,063464$	$-0,36507$
Range	$0,000383$	$0,000889$	$0,0064$	$0,856078$
Minimum	$6,12\cdot10^{-7}$	$3,68\cdot10^{-6}$	$5,22\cdot10^{-9}$	$0,0081$
Maximum	$0,000384$	$0,000893$	$0,0064$	$0,0001$
Sum	$0,001008$	$0,002043$	$0,026099$	$0,0082$
Sample size	8	17	31	25
95% Confidence interval	$0,000129$	$0,000107$	$0,00065$	$0,001015$

By the results from the upper table, we can conclude that the coefficient of filtration (average value) for the sands form all coal deposits has approximately same values, which are values in the interval $(10^{-4}, 9\cdot10^{-4})$, except for deposit Brod-Gneotino with average values of $2\cdot10^{-3}$.We can make a conclusion that the comparative analysis is made on the base of large number of data for the coefficient of filtration for all coal deposits. Therefore, this analysis can be taken with big confidence.

Table 9. Coefficient of filtration of silts (kf [cm/s)]

Silts	**Suvodol**	**Lavci**
Average value	$8,61\cdot10^{-6}$	$1,98\cdot10^{-5}$
Standard error	$2,88\cdot10^{-6}$	$8,92\cdot10^{-6}$
Median	$7,21\cdot10^{-6}$	$1,03\cdot10^{-5}$
Mode	/	/
Standard deviation	$5,76\cdot10^{-6}$	$2,18\cdot10^{-5}$
Variance	$3,32\cdot10^{-11}$	$4,77\cdot10^{-10}$

Kurtosis	$0,622432$	$-1,30632$
Skewness	$1,124843$	$0,934925$
Range	$1,3\cdot 10^{=5}$	$5,08\cdot 10^{-5}$
Minimum	$3,53\cdot 10^{-6}$	$1,7\cdot 10^{-6}$
Maximum	$1,65\cdot 10^{-5}$	$5,25\cdot 10^{-5}$
Sum	$3,44\cdot 10^{-5}$	$0,000119$
Sample size	4	6
95% Confidence interval	$9,17\cdot 10^{-6}$	$2,29\cdot 10^{-5}$

By the upper table, we can see that the coefficient of filtration (average value) for the silts from Suvodol and Lavci has almost same value with range in the interval $(10^{-6}, 9\cdot 10^{-6})$.

Table 10. Coefficient of filtration of alevrolites (kf [cm/s)]

Alevrolites	**Zvegor-Stamer**	**Lavci**
Average value	$2,51\cdot 10^{-6}$	$4,73\cdot 10^{-8}$
Standard error	$9,42\cdot 10^{-7}$	$1,15\cdot 10^{-8}$
Median	$1,09\cdot 10^{-6}$	$5,22\cdot 10^{-8}$
Mode	/	/
Standard deviation	$2,98\cdot 10^{-6}$	$2\cdot 10^{-8}$
Variance	$8,88\cdot 10^{-12}$	$3,98\cdot 10^{-16}$
Kurtosis	$0,293483$	/
Skewness	$1,206392$	$-1,03241$
Range	$8,36\cdot 10^{-6}$	$3,9\cdot 10^{-8}$
Minimum	$1,73\cdot 10^{-7}$	$2,54\cdot 10^{-8}$
Maximum	$8,53\cdot 10^{-6}$	$6,44\cdot 10^{-8}$
Sum	$2,51\cdot 10^{-5}$	$1,42\cdot 10^{-7}$
Sample size	10	3
95% Confidence interval	$2,13\cdot 10^{-6}$	$4,96\cdot 10^{-8}$

By the table, easily it can be seen that the coefficient of filtration (average value) for the alevrolites from Zvegor-Stamer and Lavci has different values, the coefficient of filtration is around $n\cdot 10^{-6}$ and for Lavci is $n\cdot 10^{-8}$, $1\le n\le 9$.

3.4. Correlation between Unit weight (γ) –dry unit weight (γ_d)

Using all available data for the dry unit weight and unit weight from all coal deposits, correlation analysis was made. At all analysis, the coefficient of correlation is greater than $0,5$. In sequel, a correlation analysis for gravels, sands, silts, clays, organic clays, alevrolites and coals will be presented.

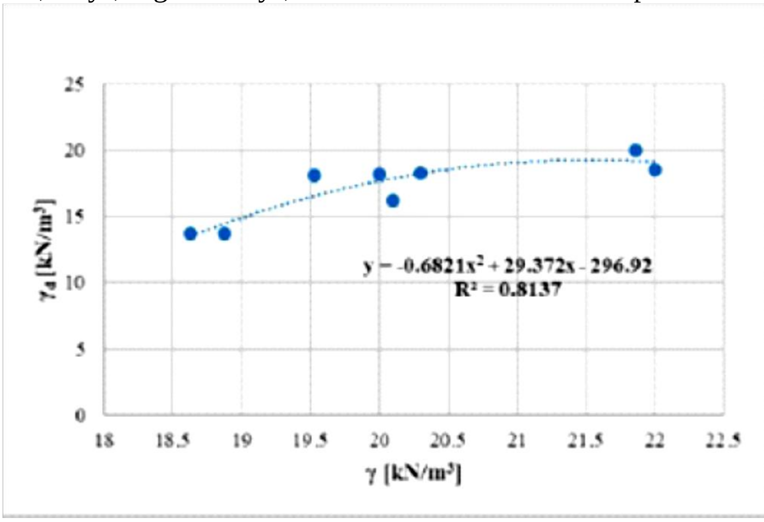

Figure 2. Correlation between unit weight and dry unit weight for gravels

From the correlation analysis for gravels, we can conclude that there is strong dependence between the parameters with coefficient of correlation ($R^2 = 0,81$) and the regression equation is given by

$$\gamma_d = -0,6821\gamma^2 + 29,372\gamma - 296,92 .$$

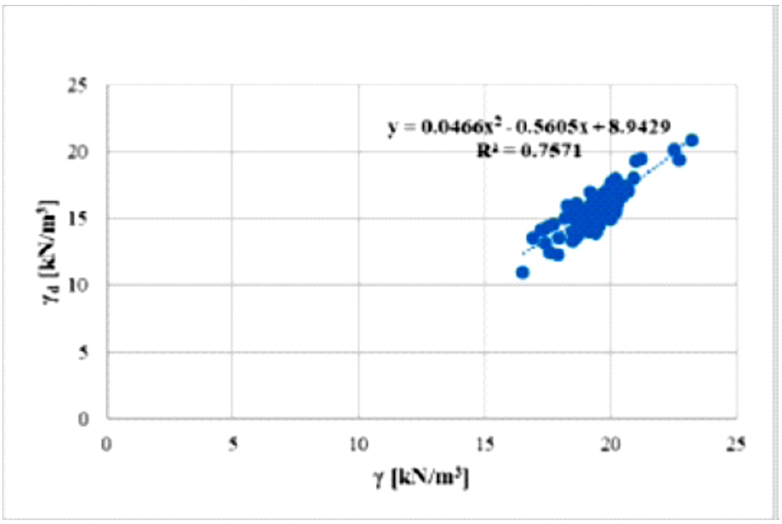

Figure 3. Correlation between unit weight and dry unit weight for sands

We can conclude that there is strong dependence for sands, between the parameters with coefficient of correlation ($R^2 = 0,76$) and regression equation given by

$$\gamma_d = 0,0466\gamma^2 - 0,5605\gamma + 8,9429.$$

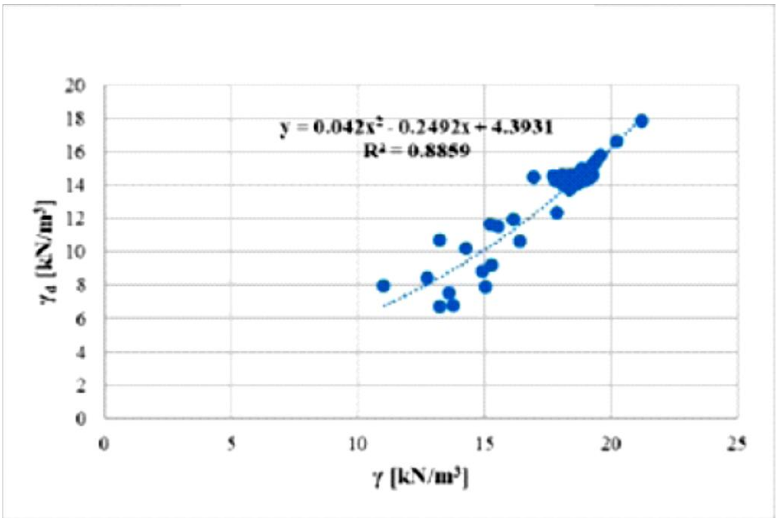

Figure 4. Correlation between unit weight and dry unit weight for silts

We can conclude that there is strong dependence for silts, between the parameters ($R^2 = 0,88$) and regression equation given by

$$\gamma_d = 0,042\gamma^2 - 0,2492\gamma + 4,3931.$$

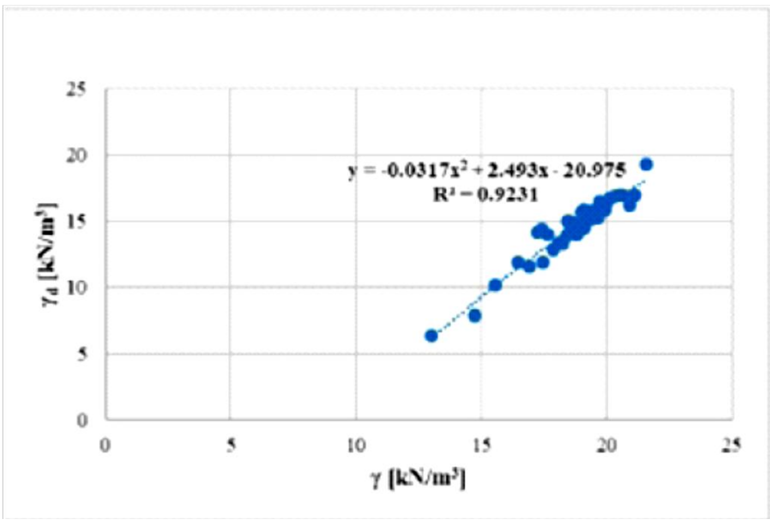

Figure 5. Correlation between unit weight and dry unit weight for clays

According to the analysis, we can conclude that for the clays there exist very strong dependence between parameters ($R^2 = 0,92$) with regression equation

$$\gamma_d = -0,0317\gamma^2 + 2,493\gamma - 20,975 .$$

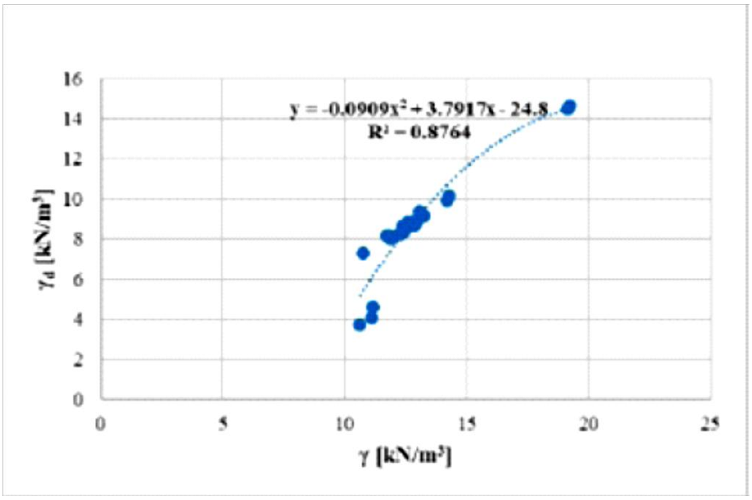

Figure 6. Correlation between unit weight and dry unit weight for coals

According to the analysis, we can conclude that there is strong dependence between parameters ($R^2 = 0,88$), and the regression is given with the equation

$$\gamma_d = -0,0909\gamma^2 + 3,7917\gamma - 24,8 .$$

3.5. Correlation between Porosity (n) – Coefficient of filtration (kf)

From the all available data for the coefficient of filtration and porosity at all coal deposits, only for the sands the regression was made. There are used twelve samples of pairs porosity and coefficient of filtration.

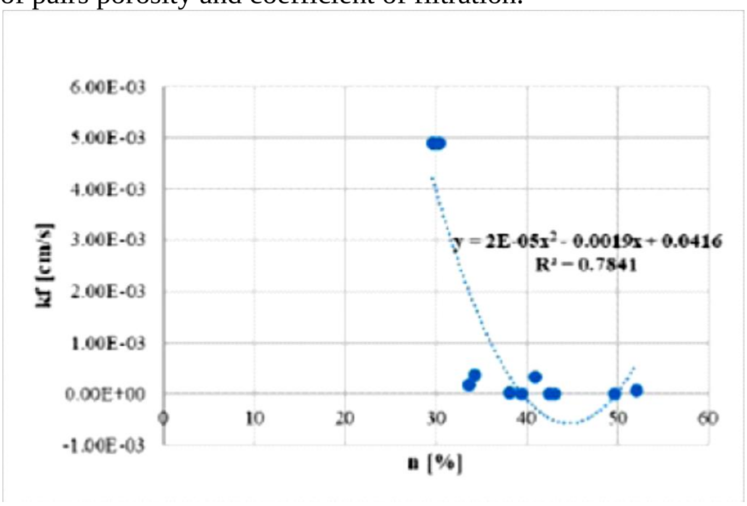

Figure 7. Correlation between porosity and coefficient of filtration for sands

We can conclude that there is strong dependence between parameters ($R^2 = 0,78$) with the regression equation

$$kf = 2 \cdot 10^{-5} n^2 - 0,0019n + 0,0416 .$$

3.6. Other attempts for correlation

Having on mind the already given dependences with coefficient of correlation greater than $0,5$ (giving the existence of some dependence between parameters), an attempt for other correlations for other geotechnical parameters was made. There was an analysis of unit weight (γ) and modulus of compressibility (Mv), but there is no correlation between these parameters, i.e. the obtained coefficient of correlation is less than $0,3$, which means that there is no apparent regression connection between these two parameters. Also, an analysis for cohesion (c) and modulus of compressibility (Mv) was made, but again the obtained result points that there is no apparent regression dependence between these two geotechnical parameters.

Another unsuccessful correlation is connections between residual angle of friction (φ_{rez}) and angle of friction (φ), as well as modulus of compressibility (Mv) and uniaxial strength (qu). In order to obtain exact and confident analysis it is very important to have large set of samples of pairs of these parameters, which in our case was absent.

4. CONCLUSIONS

Geotechnical engineering, besides in the civil engineering and environment protection, is quite important science in coal exploration, where the problems of stability of excavations, whether they are underground or surface excavations are present.

Since the surface excavation is the only way of coal exploitation in our country, it is necessary to opening of surface mines with relatively great depth, which leads to the need for design of stable slopes. In order to achieve this, is very important that detailed knowledge for geological and geotechnical profile of the considered deposit is secured.

For that purpose, the complex investigations, consisting of drilling of exploratory boreholes and field experiments, as well as laboratory tests are made. The main goal of these activities is to determine the representative geotechnical parameters for all types of materials in the deposit.

The problems given in this paper are interesting of scientific and practice point of view. The coal deposits are characterized with specific geotechnical characteristics, hence applying the approach presented in this paper give us easier determination of the geotechnical parameters, with drawing of conclusions according to authoritative statistical parameters, correlations with big enough coefficient of correlation, rejection of no logical data, efficiency in the definition of vertical geotechnical profile of the deposits. In this way, we obtain a direction in more efficient and more economical programming of future investigations and explorations of coal deposits.

Therefore, the findings of this paper should be considered very relevant and presented techniques and results can be suggested and applied in engineering practice. The approach can be upgraded with new scientific researches and practical explorations, and thus establishing geotechnical database which will enable us to give more confident correlations applicable in practical works at coal mines.

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Publisher
Union od Mathematicians of Macedonia – ARMAGANKA

Editor in chief
Prof. d-r Aleksa Malcheski

CIP - Каталогизација во публикација
Национална и универзитетска библиотека "Св. Климент Охридски", Скопје

51(082)

PROCEEDINGS of CODEMA 2020 = Зборник на трудови од CODEMA 2020. -
Skopje : Armaganka, 2021. - 375 стр. : илустр. ; 25 см

Текст на мак. и англ. јазик. - Фусноти кон текстот. - Библиографија кон трудовите

ISBN 978-608-4904-09-0

а) Математика -- Зборници

COBISS.MK-ID 53570309